# Supplement to 'The Levered Equity Risk Premium and Credit Spreads: A Unified Framework'*


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# S-I Derivation of the state-price density

In this section, we derive the state-price density shown in Proposition A1.

**Proof of Proposition A1.** Duffie and Skiadas (1994) show that the state-price density for a *general* normalized aggregator $f$ is given by

$$\pi_t = e^{\int_0^t f_v(C_s, J_s)dt} f_c\left(C_t, J_t\right), \tag{S-1}$$

where $f_c(\cdot,\cdot)$ and $f_v(\cdot,\cdot)$ are the partial derivatives of $f$ with respect to its first and second arguments, respectively, and $J$ is the value function given in (A4). The Feynman-Kac Theorem implies

$$f\left(C_t, J_{t-}\right)\big|_{\nu_{t-}=i}\, dt + E_t\left[dJ_t \middle| \nu_{t-} = i\right] = 0,\ \ i \in \{1,2\}.$$

Using Ito's Lemma we rewrite the above equation as

$$0 = f\left(C, J_i\right) + C J_{i,C} g_i + \frac{1}{2} C^2 J_{i,CC} \sigma_{C,i}^2 + \lambda_i \left(J_j - J_i\right), \tag{S-2}$$

for $i, j \in \{1,2\}$, $j \neq i$. We guess and verify that $J = h(CV)$, where $V_i$ satisfies the nonlinear equation system

$$0 = \beta u\left(V_i^{-1}\right) + g_i - \frac{1}{2}\gamma\sigma_{C,i}^2 + \lambda_i \left(\frac{\left(V_j/V_i\right)^{1-\gamma} - 1}{1-\gamma}\right),\ i, j \in \{1,2\},\ j \neq i. \tag{S-3}$$

Substituting (A3) into (S-1) and simplifying gives

$$\pi_t = \beta e^{-\beta\int_0^t \left[1 + \left(\gamma - \frac{1}{\psi}\right) u\left(V_s^{-1}\right)\right]dt} C_t^{-\gamma} V_t^{-\left(\gamma - \frac{1}{\psi}\right)}. \tag{S-4}$$

When $\psi = 1$, the above equation gives the second expression in (A5). We rewrite (S-3) as

$$\beta\left[1 + \left(\gamma - \frac{1}{\psi}\right) u\left(V_i^{-1}\right)\right] = \overline{r}_i - \left(\gamma - \frac{1}{\psi}\right)\lambda_i \left(\frac{\left(V_j/V_i\right)^{1-\gamma} - 1}{1-\gamma}\right) - \left[\gamma g_i - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma_{C,i}^2\right],\ i, j \in \{1,2\},\ j \neq i, \tag{S-5}$$

where $\overline{r}_i$ is given in (A7). Setting $\psi = 1$ in (S-5) gives (A9). To derive the first expression in (A5) from (S-4) we prove that

$$V_i = \left(\beta p_{C,i}\right)^{\frac{1}{1-\frac{1}{\psi}}},\ \psi \neq 1. \tag{S-6}$$

We proceed by considering the optimization problem for the representative agent. She chooses her optimal consumption, $C^*$, and risky asset portfolio, $\varphi$, to maximize her expected utility

$$J_t^* = \sup_{C^*,\varphi} E_t \int_t^\infty f\left(C_t^*, J_t^*\right) dt.$$

Observe that $J^*$ depends on optimal consumption-portfolio choice, whereas the $J$ defined previously in (A8) depends on exogenous aggregate consumption. The optimization is carried out subject to the dynamic budget constraint, which we now describe. If the agent consumes at the rate, $C^*$, invests a proportion, $\varphi$, of her remaining financial wealth in the claim on aggregate consumption (the risky asset), and puts the remainder in the locally risk-free asset, then her financial wealth, $W$, evolves according to the dynamic budget constraint:

$$\frac{dW_t}{W_{t-}} = \varphi_{t-}\left(dR_{C,t} - r_{t-}dt\right) + r_{t-}dt - \frac{C^*_{t-}}{W_{t-}}dt,$$

where $dR_{C,t}$ is the cumulative return on the claim to aggregate consumption. We define $N_{i,t}$ as the Poisson process which jumps upward by one whenever the state of the economy switches from $i$ to $j \neq i$. The compensated version of this process is the Poisson martingale

$$N^P_{i,t} = N_{i,t} - \lambda_i t.$$

It follows from applying Ito's Lemma to $P = p_C C$ that the cumulative return on the claim to aggregate consumption is

$$dR_{C,t} = \frac{dP_t + C_t dt}{P_{t-}} = \mu_{R_C,t-}dt + \sigma_{C,t-}dB_{C,t} + \sigma^P_{R_C,t-}dN^P_{\nu_{t-},t},$$

where

$$\begin{aligned}
\mu_{R_C,t-}\big|_{\nu_{t-}=i} &= \mu_{R_C,i} = g_i + \frac{1}{2}\sigma^2_{C,i} + \lambda_i\left(\frac{p_{C,j}}{p_{C,i}} - 1\right) + \frac{1}{p_{C,i}}, \\
\sigma_{C,t-}\big|_{\nu_{t-}=i} &= \sigma_{C,i}, \\
\sigma^P_{R_C,t-}\big|_{\nu_{t-}=i} &= \sigma^P_{R_C,i} = \frac{p_{C,j}}{p_{C,i}} - 1,
\end{aligned}$$

for $i \in \{1,2\}$, $j \neq i$. The total volatility of the return to holding the consumption claim, when the current state is $i$, is given by

$$\sigma_{R_C,i} = \sqrt{\sigma^2_{C,i} + \lambda_i\left(\sigma^P_{R_C,i}\right)^2}.$$

Note that $C^*$ is the consumption to be chosen by the agent, i.e. it is a control, and at this stage we cannot rule out the possibility that it jumps with the state of the economy. In contrast, $C$ is aggregate consumption, and since it is continuous, its left and right limits are equal, i.e. $C_{t-} = C_t$.

The system of Hamilton-Jacobi-Bellman partial differential equations for the agent's optimization problem is

$$\sup_{C^*,\varphi} f\left(C^*_{t-}, J^*_{t-}\right)\big|_{\nu_{t-}=i}dt + E_t\left[dJ^*_t \,|\, \nu_{t-} = i\right] = 0,\ \ i \in \{1,2\}.$$

Applying Ito's Lemma to $J^*_t = J^*\left(W_t, \nu_t\right)$ allows us to write the above equation as

$$\begin{aligned}
0 &= \sup_{C^*_i,\varphi_i} f\left(C^*_i, J^*_i\right) + W_i J^*_{i,W}\left(\varphi_i\left(\mu_{R_C,i} - r_i\right) + r_i - \frac{C^*_i}{W_i}\right) + \frac{1}{2}W^2_i J^*_{i,WW}\varphi^2_i\sigma^2_{R_C,i} + \\
&\quad + \lambda_i\left(J^*_j - J^*_i\right),\ i \in \{1,2\},\ j \neq i.
\end{aligned}$$

We guess and verify that $J^*_t = h\left(W_t F_t\right)$, where $F_i$ satisfies the nonlinear equation system

$$0 = \sup_{C^*_i,\varphi_i} \beta u\left(\frac{C^*_i}{W_i F_i}\right) + \left(\varphi_i\left(\mu_{R_C,i} - r_i\right) + r_i - \frac{C^*_i}{W_i}\right) - \frac{1}{2}\gamma\varphi^2_i\sigma^2_{R_C,i} + \lambda_i\left(\frac{\left(F_j/F_i\right)^{1-\gamma} - 1}{1-\gamma}\right),\ i \in \{1,2\},\ j \neq i.$$

From the first order conditions of the above equations, we obtain the optimal consumption and portfolio policies:

$$\begin{aligned} C_i^* &= \beta^{\psi} F_i^{-(\psi-1)} W_i, \; i \in \{1,2\}, \\ \varphi_i &= \frac{1}{\gamma} \frac{\mu_{R_C,i} - r_i}{\sigma^2_{R_C,i}}, \; i \in \{1,2\}. \end{aligned}$$

The market for the consumption good must clear, so $\varphi_i = 1$, $W_i = P_i$, $C_i^* = C$ (and thus $J = J^*$). Note that this forces the optimal portfolio proportion to be one and the optimal consumption policy to be continuous. Hence

$$\mu_{R_C,i} - r_i = \gamma \sigma^2_{R_C,i},$$

and

$$p_{C,i} = \beta^{-\psi} F_i^{1-\psi}. \tag{S-7}$$

The above equation implies that for $\psi = 1$, $p_{C,i} = 1/\beta$. The equality, $J = J^*$, implies that $CV_i = WF_i$. Hence, $F_i = p_{C,i}^{-1} V_i$. Using this equation to eliminate $F_i$ from (S-7) gives (S-6). Substituting (S-6) into (S-4) and (S-5) gives the expression in (A5) for $\psi \neq 1$ and (A6). ■

# S-II Perpetual Arrow-Debreu default claims–static capital structure

**Proposition S-1** *Under static capital structure, the price of the perpetual Arrow-Debreu default claim,* $q_{D,ij} = \lim_{T-t\to\infty} q_{D,ij,T-t}$*, is given by*

$$q_{D,11} = \begin{cases} \sum_{m=1}^{2} h_{11,m} X^{k_m}, & X > X_{D,1}; \\ 1, & X_{D,2} < X \leq X_{D,1}; \\ 1, & X \leq X_{D,2}, \end{cases} \qquad q_{D,12} = \begin{cases} \sum_{m=1}^{2} h_{12,m} X^{k_m}, & X > X_{D,1}; \\ 0, & X_{D,2} < X \leq X_{D,1}; \\ 0, & X \leq X_{D,2}, \end{cases}$$

$$q_{D,21} = \begin{cases} \sum_{m=1}^{2} h_{11,m} \epsilon(k_m) X^{k_m}, & X > X_{D,1}; \\ \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_{21}} + \sum_{m=1}^{2} s_{1,m} X^{j_m}, & X_{D,2} < X \leq X_{D,1}; \\ 0, & X \leq X_{D,2}, \end{cases} \qquad q_{D,22} = \begin{cases} \sum_{m=1}^{2} h_{12,m} \epsilon(k_m) X^{k_m}, & X > X_{D,1}; \\ \sum_{m=1}^{2} s_{2,m} X^{j_m}, & X_{D,2} < X \leq X_{D,1}; \\ 1, & X \leq X_{D,2}, \end{cases}, \tag{S-8}$$

*where* $k_1 < k_2 < 0$ *are the negative roots of the quartic (S-15),* $\epsilon(k)$ *is defined by (S-16),* $j_1 < j_2$ *are the roots of the quadratic (S-17) and* $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$ *are given by (S-18).*

**Proof of Proposition S-1.** Letting $T - t \to \infty$ in $q_{D,ij,T-t}$, we obtain the perpetual Arrow-Debreu Default Claims, $q_{D,ij}$. The no-arbitrage principle gives (7), which using Ito's Lemma can be rewritten as the following ordinary differential-equation system:

$$\frac{dq_{D,ij}}{dX} \widehat{\theta}_i X + \frac{1}{2} \frac{d^2 q_{D,ij}}{dX^2} \sigma^2_{X,i} X^2 + \widehat{\lambda}_i (q_{D,kj} - q_{D,ij}) = r_i q_{D,ij}, \; i,j \in \{1,2\}, k \neq i, \tag{S-9}$$

where

$$\sigma_{X,i} = \sqrt{(\sigma_X^{id})^2 + (\sigma_{X,i}^s)^2} \tag{S-10}$$

is total earnings growth volatility in state $i$ and

$$\widehat{\theta}_i = \theta_i - \gamma \rho_{XC,i} \sigma^s_{X,i} \sigma_{C,i}$$

is the risk-neutral earnings growth rate in state $i$. The definitions of the payoffs of the Arrow-Debreu default claims give us the following boundary conditions:

$$q_{D,ij}(X) = \begin{cases} 1, & j = i,\ X \le X_{D,i} \\ 0, & j \ne i,\ X \le X_{D,i} \end{cases}. \tag{S-11}$$

Value-matching and smooth-pasting give us the remaining boundary conditions: for $j \in \{1,2\}$

$$\begin{aligned} \lim_{X \downarrow X_{D,j}} q_{D,2j} &= \lim_{X \uparrow X_{D,j}} q_{D,2j}, \\ \lim_{X \downarrow X_{D,j}} q'_{D,2j} &= \lim_{X \uparrow X_{D,j}} q'_{D,2j}. \end{aligned}$$

Expressing (S-9) in matrix form gives:

$$\left(\frac{1}{2}\begin{bmatrix} \sigma^2_{1,X} & 0 \\ 0 & \sigma^2_{2,X} \end{bmatrix} X^2 \frac{d^2}{dX^2} + \begin{bmatrix} \widehat{\theta}_1 & 0 \\ 0 & \widehat{\theta}_2 \end{bmatrix} X \frac{d}{dX} - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} + \begin{bmatrix} -\widehat{\lambda}_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 \end{bmatrix}\right) \begin{bmatrix} q_{D,11} & q_{D,12} \\ q_{D,21} & q_{D,22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{S-12}$$

From (S-11) it follows that

$$q_{D,ij}\big|_{X=X_{D,i}} = \begin{cases} 1, & j = i; \\ 0, & j \ne i. \end{cases} \tag{S-13}$$

We first solve (S-12) subject to the conditions above for the region $X > X_{D,1}$. We seek solutions of the form $q_{D,ij} = h_{ij} X^k$, $i, j \in \{1,2\}$, where

$$\left(\frac{1}{2}\begin{bmatrix} \sigma^2_{X,1} & 0 \\ 0 & \sigma^2_{X,2} \end{bmatrix} k(k-1) + \begin{bmatrix} \widehat{\theta}_1 & 0 \\ 0 & \widehat{\theta}_2 \end{bmatrix} k + \begin{bmatrix} -\widehat{\lambda}_1 - r_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 - r_2 \end{bmatrix}\right) \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{S-14}$$

A solution of the above equation exists if

$$\det\left(\frac{1}{2}\begin{bmatrix} \sigma^2_{X,1} & 0 \\ 0 & \sigma^2_{X,2} \end{bmatrix} k(k-1) + \begin{bmatrix} \widehat{\theta}_1 & 0 \\ 0 & \widehat{\theta}_2 \end{bmatrix} k + \begin{bmatrix} -\widehat{\lambda}_1 - r_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 - r_2 \end{bmatrix}\right) = 0,$$

i.e. $k$ is a root of the quartic polynomial

$$\left[\frac{1}{2}\sigma^2_{X,1} k(k-1) + \widehat{\theta}_1 k - \left(\widehat{\lambda}_1 + r_1\right)\right]\left[\frac{1}{2}\sigma^2_{X,2} k(k-1) + \widehat{\theta}_2 k - \left(\widehat{\lambda}_2 + r_2\right)\right] - \widehat{\lambda}_2 \widehat{\lambda}_1 = 0, \tag{S-15}$$

which is the characteristic function of (S-12). The above quartic has 4 distinct real roots, two of which are positive, provided that $\sigma_{X,i}$, $r_i$, $\widehat{\lambda}_i > 0$ for $i \in \{1,2\}$ (see Guo (2001)). Therefore the general solution of (S-14) is

$$q_{D,ij} = \sum_{m=1}^{4} h_{ij,m} X^{k_m},$$

where $k_m$ is the $m$'th root (ranked in order of increasing size, accounting for sign) of (S-15). To ensure that $q_{D,ij}$, $i, j \in \{1,2\}$ are finite as $X \to \infty$, we set $h_{ij,3} = h_{ij,4} = 0$, $i, j \in \{1,2\}$, so we use only the two negative roots: $k_1 < k_2 < 0$. From equation (S-14), it follows that

$$\frac{h_{21,m}}{h_{11,m}} = \frac{h_{22,m}}{h_{12,m}} = \epsilon(k_m),\ m \in \{1,2\},$$

where

$$\epsilon(k) = -\frac{\widehat{\lambda}_2}{\frac{1}{2}\sigma_{X,2}^2 k(k-1) + \widehat{\theta}_2 k - \left(\widehat{\lambda}_2 + r_2\right)} = -\frac{\frac{1}{2}\sigma_{X,1}^2 k(k-1) + \widehat{\theta}_1 k - \left(\widehat{\lambda}_1 + r_1\right)}{\widehat{\lambda}_1}. \tag{S-16}$$

Therefore

$$\begin{aligned} q_{D,1j} &= \sum_{m=1}^{2} h_{1j,m} X^{k_m},\ j \in \{1,2\}, \\ q_{D,2j} &= \sum_{m=1}^{2} h_{1j,m} \epsilon(k_m) X^{k_m},\ j \in \{1,2\}. \end{aligned}$$

We now solve (S-12) subject to the relevant boundary conditions for the region $X_{D,2} < X \leq X_{D,1}$. We know $q_{D,11} = 1$, $q_{D,12} = 0$. Therefore

$$\left(\frac{1}{2}\sigma_{X,2}^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d^2}{dX^2} + \widehat{\theta}_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d}{dX} - \left(\widehat{\lambda}_2 + r_2\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} q_{D,21,t} \\ q_{D,22,t} \end{bmatrix} + \begin{bmatrix} \widehat{\lambda}_2 \\ 0 \end{bmatrix} = 0.$$

We can show (using the same method we use to solve (S-12)) that the general solution of the above equation is

$$\begin{aligned} q_{D,21} &= \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2} + s_{1,1} X^{j_1} + s_{1,2} X^{j_2}, \\ q_{D,22} &= s_{2,1} X^{j_1} + s_{2,2} X^{j_2}, \end{aligned}$$

where $j_m$, $m \in \{1,2\}$, are the roots of the quadratic

$$\frac{1}{2}\sigma_{X,2}^2 j(j-1) + \widehat{\theta}_2 j - \left(\widehat{\lambda}_2 + r_2\right) = 0, \tag{S-17}$$

such that $j_1 < j_2$. Expressions for Arrow-Debreu default claims are summarized in (S-8). To find the 8 constants: $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$, we use the following 8 boundary conditions:

$$\begin{aligned} q_{D,11}|_{X=X_{D,1}} &= 1, \qquad q_{D,12}|_{X=X_{D,1}} = 0, \\ \lim_{X \uparrow X_{D,1}} q_{D,21} &= \lim_{X \downarrow X_{D,1}} q_{D,21}, \lim_{X \uparrow X_{D,1}} q_{D,22} = \lim_{X \downarrow X_{D,1}} q_{D,22}, \\ \lim_{X \uparrow X_{D,1}} q'_{D,21} &= \lim_{X \downarrow X_{D,1}} q'_{D,21}, \lim_{X \uparrow X_{D,1}} q'_{D,22} = \lim_{X \downarrow X_{D,1}} q'_{D,22}, \\ q_{D,21}|_{X=X_{D,2}} &= 0, \qquad q_{D,22}|_{X=X_{D,2}} = 1. \end{aligned}$$

The 8 boundary conditions give 8 linear equations, which can be written in matrix form as

$$\begin{pmatrix} X_{D,1}^{k_1} & X_{D,1}^{k_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_{D,1}^{k_1} & X_{D,1}^{k_2} & 0 & 0 & 0 & 0 \\ \epsilon(k_1)X_{D,1}^{k_1} & \epsilon(k_2)X_{D,1}^{k_2} & 0 & 0 & -X_{D,1}^{j_1} & -X_{D,1}^{j_2} & 0 & 0 \\ 0 & 0 & \epsilon(k_1)X_{D,1}^{k_1} & \epsilon(k_2)X_{D,1}^{k_2} & 0 & 0 & -X_{D,1}^{j_1} & -X_{D,1}^{j_2} \\ k_1\epsilon(k_1)X_{D,1}^{k_1} & k_2\epsilon(k_2)X_{D,1}^{k_2} & 0 & 0 & -j_1X_{D,1}^{j_1} & -j_2X_{D,1}^{j_2} & 0 & 0 \\ 0 & 0 & k_1\epsilon(k_1)X_{D,1}^{k_1} & k_2\epsilon(k_2)X_{D,1}^{k_2} & 0 & 0 & -j_1X_{D,1}^{j_1} & -j_2X_{D,1}^{j_2} \\ 0 & 0 & 0 & 0 & X_{D,2}^{j_1} & X_{D,2}^{j_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_{D,2}^{j_1} & X_{D,2}^{j_2} \end{pmatrix} \begin{pmatrix} h_{11,1} \\ h_{11,2} \\ h_{12,1} \\ h_{12,2} \\ s_{1,1} \\ s_{1,2} \\ s_{2,1} \\ s_{2,2} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \frac{\widehat{\lambda}_{21}}{r_2+\widehat{\lambda}_{21}} \\ 0 \\ 0 \\ 0 \\ -\frac{\widehat{\lambda}_{21}}{r_2+\widehat{\lambda}_{21}} \\ 1 \end{pmatrix} \tag{S-18}$$

The above equation system can be solved in closed-form to give $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$.

We obtain $\{\widehat{p}_{D,ij}\}_{i,j\in\{1,2\}}$ and $\{p_{D,ij}\}_{i,j\in\{1,2\}}$, by setting $r_1 = r_2 = 0$, and $r_1 = r_2 = 0$, $\gamma = 1/\psi = 0$, respectively. Then we can compute the perpetual risk and time adjustments via $\mathcal{R}_{ij} = \frac{\hat{p}_{D,ij}}{p_{D,ij}}$ and $\mathcal{T}_{ij} = \frac{q_{D,ij}}{\hat{p}_{D,ij}}$. ■

## S-III Perpetual Arrow-Debreu restructuring/default claims–dynamic capital structure

**Proposition S-2** *If $X_{U,2} < X_{U,1}$, then the perpetual Arrow-Debreu restructuring claim is given by*

$$q_{11,U} = \begin{cases} 1 & X > X_{U,1}; \\ \sum_{m=1}^{2} g_{11,m} X^{l_m}, & X_{U,2} < X \le X_{U,1}; \\ \sum_{m=1}^{4} h_{11,m} X^{k_m}, & X_{D,1} < X \le X_{U,2}; \\ 0, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}. \end{cases} \quad q_{12,U} = \begin{cases} 0 & X > X_{U,1}; \\ \sum_{m=1}^{2} g_{12,m} X^{l_m} + \frac{\widehat{\lambda}_{12}}{r_1 + \widehat{\lambda}_{12}}, & X_{U,2} < X \le X_{U,1}; \\ \sum_{m=1}^{4} h_{12,m} X^{k_m}, & X_{D,1} < X \le X_{U,1}; \\ 0, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}. \end{cases}$$

$$q_{21,U} = \begin{cases} 0 & X > X_{U,1}; \\ 0 & X_{U,2} < X \le X_{U,1}; \\ \sum_{m=1}^{4} h_{11,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \le X_{U,1}; \\ \sum_{m=1}^{2} s_{21,m} X^{j_m}, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}; \end{cases} \quad q_{22,U} = \begin{cases} 1 & X > X_{U,2}; \\ 1 & X_{U,1} < X \le X_{U,2}; \\ \sum_{m=1}^{4} h_{12,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \le X_{U,2}; \\ \sum_{m=1}^{2} s_{22,m} X^{j_m}, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}, \end{cases} \tag{S-19}$$

*where $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$ are determined by (S-23), and the perpetual Arrow-Debreu default claim is given by*

$$q_{11,D} = \begin{cases} 0 & X > X_{U,1} \\ \sum_{m=1}^{2} g_{11,m} X^{l_m}, & X_{U,2} < X \le X_{U,1}; \\ \sum_{m=1}^{4} h_{11,m} X^{k_m}, & X_{D,1} < X \le X_{U,2}; \\ 1, & X_{D,2} < X \le X_{D,1}; \\ 1, & X \le X_{D,2}, \end{cases} \quad q_{12,D} = \begin{cases} 0 & X > X_{U,1}; \\ \sum_{m=1}^{2} g_{12,m} X^{l_m}, & X_{U,2} < X \le X_{U,1}; \\ \sum_{m=1}^{4} h_{12,m} X^{k_m}, & X_{D,1} < X \le X_{U,1}; \\ 0, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}, \end{cases}$$

$$q_{21,D} = \begin{cases} 0 & X > X_{U,1}; \\ 0 & X_{U,2} < X \le X_{U,1}; \\ \sum_{m=1}^{4} h_{11,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \le X_{U,1}; \\ \sum_{m=1}^{2} s_{21,m} X^{j_m} + \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2}, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}, \end{cases} \quad q_{22,D} = \begin{cases} 0 & X > X_{U,2}; \\ 0 & X_{U,1} < X \le X_{U,2}; \\ \sum_{m=1}^{4} h_{12,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \le X_{U,2}; \\ \sum_{m=1}^{2} s_{22,m} X^{j_m}, & X_{D,2} < X \le X_{D,1}; \\ 1, & X \le X_{D,2}, \end{cases} \tag{S-20}$$

*where $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$ are determined by (S-24). If $X_{U,1} < X_{U,2}$, then the perpetual Arrow-Debreu restructuring claim is given by*

$$q_{11,U} = \begin{cases} 1 & X > X_{U,2}; \\ 1, & X_{U,1} < X \le X_{U,2}; \\ \sum_{m=1}^{4} h_{11,m} X^{k_m}, & X_{D,1} < X \le X_{U,1}; \\ 0, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}, \end{cases} \quad q_{12,U} = \begin{cases} 0 & X > X_{U,2}; \\ 0, & X_{U,1} < X \le X_{U,2}; \\ \sum_{m=1}^{4} h_{12,m} X^{k_m}, & X_{D,1} < X \le X_{U,1}; \\ 0, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}, \end{cases}$$

$$q_{21,U} = \begin{cases} 0 & X > X_{U,2} \\ \frac{\widehat{\lambda}_1}{\widehat{\lambda}_2 + r_2} + \sum_{m=1}^{2} g_{21,m} X^{j_m} & X_{U,1} < X \le X_{U,2}; \\ \sum_{m=1}^{4} h_{11,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \le X_{U,1}; \\ \sum_{m=1}^{2} s_{21,m} X^{l_m}, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}; \end{cases} \quad q_{22,U} = \begin{cases} 1 & X > X_{U,2}; \\ \sum_{m=1}^{2} g_{22,m} X^{j_m} & X_{U,1} < X \le X_{U,2}; \\ \sum_{m=1}^{4} h_{12,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \le X_{U,2}; \\ \sum_{m=1}^{2} s_{22,m} X^{j_m}, & X_{D,2} < X \le X_{D,1}; \\ 0, & X \le X_{D,2}, \end{cases} \tag{S-21}$$

*where $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$ are determined by (S-25), and the perpetual Arrow-Debreu default claim is given by*

$$q_{11,D} = \begin{cases} 0 & X > X_{U,2}; \\ 0, & X_{U,1} < X \leq X_{U,2}; \\ \sum_{m=1}^{4} h_{11,m} X^{k_m}, & X_{D,1} < X \leq X_{U,1}; \\ 1, & X_{D,2} < X \leq X_{D,1}; \\ 1, & X \leq X_{D,2}, \end{cases} \quad q_{12,U} = \begin{cases} 0 & X > X_{U,2}; \\ 0, & X_{U,1} < X \leq X_{U,2}; \\ \sum_{m=1}^{4} h_{12,m} X^{k_m}, & X_{D,1} < X \leq X_{U,1}; \\ 0, & X_{D,2} < X \leq X_{D,1}; \\ 0, & X \leq X_{D,2}, \end{cases}$$

$$q_{21,U} = \begin{cases} 0 & X > X_{U,2}; \\ \sum_{m=1}^{2} g_{21,m} X^{k_m} & X_{U,1} < X \leq X_{U,2}; \\ \sum_{m=1}^{4} h_{11,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \leq X_{U,1}; \\ \sum_{m=1}^{2} s_{21,m} X^{j_m} + \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2}, & X_{D,2} < X \leq X_{D,1}; \\ 0, & X \leq X_{D,2}, \end{cases} \quad q_{22,U} = \begin{cases} 1 & X > X_{U,2}; \\ \sum_{m=1}^{2} g_{22,m} X^{k_m} & X_{U,1} < X \leq X_{U,2}; \\ \sum_{m=1}^{4} h_{12,m} \epsilon(k_m) X^{k_m}, & X_{D,1} < X \leq X_{U,2}; \\ \sum_{m=1}^{2} s_{22,m} X^{j_m}, & X_{D,2} < X \leq X_{D,1}; \\ 0, & X \leq X_{D,2}, \end{cases} . \tag{S-22}$$

*where $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$ are determined by (S-26).*

**Proof of Proposition S-2.** We consider the cases $X_{U,2} < X_{U,1}$ and $X_{U,1} < X_{U,2}$ separately. When $X_{U,2} < X_{U,1}$, we must solve for Arrow-Debreu default and restructuring claims in the following 5 regions:

1. $X > X_{U,1}$
2. $X_{U,2} \leq X \leq X_{U,1}$
3. $X_{D,1} \leq X \leq X_{U,2}$
4. $X_{D,2} \leq X \leq X_{D,1}$
5. $X \leq X_{D,2}$

The time-$t$ price of the perpetual Arrow-Debreu restucturing claim which pays out a unit of consumption at restructuring if restructuring occurs in state $j$, conditional on the current state being $i$ satisfies the basic asset pricing equation

$$E_t^{\mathbb{Q}} \left[ dq_{U,ij} - r_i q_{U,ij} dt | \nu_t = i \right].$$

Therefore,

$$\frac{1}{2} X^2 \sigma_{X,i}^2 \frac{d^2 q_{U,ij}}{dX^2} + X \hat{\theta}_i \frac{dq_{U,ij}}{dX} + \widehat{\lambda}_i (q_{U,kj} - q_{U,ij}) - r_i q_{U,ij} = 0,$$

subject to relevant boundary conditions. In the first of the above regions, restructuring is immediate whatever the current state is. Therefore $q_{U,11} = q_{U,22} = 1$ and $q_{U,12} = q_{U,21} = 0$. In the second region, restructuring is immediate if the current state is 2. Therefore, $q_{U,22} = 1$ and $q_{U,21} = 0$. To find $q_{U,11}$ and $q_{U,12}$, we must solve the ode's:

$$\frac{1}{2} X^2 \sigma_{X,1}^2 \frac{d^2 q_{U,11}}{dX^2} + X \hat{\theta}_1 \frac{dq_{U,11}}{dX} - (r_1 + \widehat{\lambda}_1) q_{U,11} = 0,$$

and

$$\frac{1}{2} X^2 \sigma_{X,1}^2 \frac{d^2 q_{U,12}}{dX^2} + X \hat{\theta}_1 \frac{dq_{U,12}}{dX} + \widehat{\lambda}_1 (1 - q_{U,12}) - r_1 q_{U,12} = 0.$$

Therefore,

$$\left(\frac{1}{2}\sigma_{X,1}^2\begin{bmatrix}1&0\\0&1\end{bmatrix}\frac{d^2}{dX^2}+\widehat{\theta}_1\begin{bmatrix}1&0\\0&1\end{bmatrix}\frac{d}{dX}-\left(\widehat{\lambda}_1+r_1\right)\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\begin{bmatrix}q_{U,11}\\q_{U,12}\end{bmatrix}+\begin{bmatrix}0\\\widehat{\lambda}_1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix},$$

The general solutions of the above ode's are

$$\begin{aligned}q_{U,11} &= g_{11}X^{l_1}+g_{12}X^{l_2},\\ q_{U,12} &= g_{21}X^{l_1}+g_{22}X^{l_2}+\frac{\widehat{\lambda}_1}{r_1+\widehat{\lambda}_1},\end{aligned}$$

where $l_1 < l_2$ are the roots of the quadratic

$$\frac{1}{2}\sigma_{X,1}^2 l\,(l-1)+\widehat{\theta}_1 l-\left(\widehat{\lambda}_1+r_1\right)=0.$$

In the third region

$$\left(\frac{1}{2}\begin{bmatrix}\sigma_{1,X}^2&0\\0&\sigma_{2,X}^2\end{bmatrix}X^2\frac{d^2}{dX^2}+\begin{bmatrix}\widehat{\theta}_1&0\\0&\widehat{\theta}_2\end{bmatrix}X\frac{d}{dX}-\begin{bmatrix}r_1&0\\0&r_2\end{bmatrix}+\begin{bmatrix}-\widehat{\lambda}_1&\widehat{\lambda}_1\\\widehat{\lambda}_2&-\widehat{\lambda}_2\end{bmatrix}\right)\begin{bmatrix}q_{U,11}&q_{U,12}\\q_{U,21}&q_{U,22}\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}.$$

The above ode system has the following general solution

$$\begin{aligned}q_{U,1j} &= \sum_{m=1}^{4}h_{1j,m}X^{k_m},\\ q_{U,2j} &= \sum_{m=1}^{4}\epsilon(k_m)h_{1j,m}X^{k_m},\end{aligned}$$

where $k_1 < k_2 < 0 < k_3 < k_4$ are the roots of the quartic (S-15) and $\epsilon\left(k\right)$ is defined by (S-16).

In the fourth region default is immediate if the state is 1, but not if the state is 2. Therefore, $q_{U,11} = q_{U,12} = 0$ and

$$\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{U,2j}}{dX^2}+X\hat{\theta}_2\frac{dq_{U,2j}}{dX}-(r_2+\widehat{\lambda}_2)q_{U,2j}=0.$$

The general solution of the above ode is

$$q_{U,2j}=\sum_{m=1}^{2}s_{2j,m}X^{j_m}.$$

where $j_1 < j_2$ are roots of the quadratic (S-17).

In the fifth region, default is immediate in both states, and so $q_{U,11} = q_{U,12} = q_{U,21} = q_{U,22} = 0$. Therefore, in summary we have (S-19), where the 16 constants, $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$, are the solutions of the 16 simultaneous linear equations, which arise from the 16 boundary conditions:

$$\begin{aligned}
&q_{U,11}|_{X\downarrow X_{U,1}} = q_{U,11}|_{X\uparrow X_{U,1}}\,,\ q_{U,21}|_{X\downarrow X_{U,2}} = q_{U,21}|_{X\uparrow X_{U,2}}\,, q_{U,11}|_{X\downarrow X_{D,1}} = q_{U,11}|_{X\uparrow X_{D,1}}\,,\ q_{U,21}|_{X\downarrow X_{D,2}} = q_{U,21}|_{X\uparrow X_{D,2}}\,,\\
&q_{U,11}|_{X\downarrow X_{U,2}} = q_{U,11}|_{X\uparrow X_{U,2}}\,,\ q'_{U,11}|_{X\downarrow X_{U,2}} = q'_{U,11}|_{X\uparrow X_{U,2}}\,, q_{U,21}|_{X\downarrow X_{D,1}} = q_{U,21}|_{X\uparrow X_{D,1}}\,,\ q'_{U,21}|_{X\downarrow X_{D,1}} = q'_{U,21}|_{X\uparrow X_{D,1}}\,,\\
&q_{U,12}|_{X\downarrow X_{U,1}} = q_{U,12}|_{X\uparrow X_{U,1}}\,,\ q_{U,22}|_{X\downarrow X_{U,2}} = q_{U,22}|_{X\uparrow X_{U,2}}\,, q_{U,12}|_{X\downarrow X_{D,1}} = q_{U,12}|_{X\uparrow X_{D,1}}\,,\ q_{U,22}|_{X\downarrow X_{D,2}} = q_{U,22}|_{X\uparrow X_{D,2}}\,,\\
&q_{U,12}|_{X\downarrow X_{U,2}} = q_{U,12}|_{X\uparrow X_{U,2}}\,,\ q'_{U,12}|_{X\downarrow X_{U,2}} = q'_{U,12}|_{X\uparrow X_{U,2}}\,, q_{U,22}|_{X\downarrow X_{D,1}} = q_{U,22}|_{X\uparrow X_{D,1}}\,,\ q'_{U,22}|_{X\downarrow X_{D,1}} = q'_{U,22}|_{X\uparrow X_{D,1}}\,.
\end{aligned} \tag{S-23}$$

We now solve for the Arrow-Debreu default claim. The time-$t$ price of the perpetual Arrow-Debreu default claim which pays out a unit of consumption at default if default occurs in state $j$, conditional on the current state being $i$ satisfies the basic asset pricing equation

$$E_t^{\mathbb{Q}}\left[dq_{D,ij} - r_i q_{D,ij} dt | \nu_t = i\right].$$

Therefore,

$$\frac{1}{2}X^2\sigma_{X,i}^2\frac{d^2q_{D,ij}}{dX^2} + X\hat{\theta}_i\frac{dq_{D,ij}}{dX} + \widehat{\lambda}_i(q_{D,kj} - q_{D,ij}) - r_i q_{D,ij} = 0,$$

subject to relevant boundary conditions. In the first region, restructuring is immediate whatever the current state is. Therefore $q_{D,11} = q_{D,12} = q_{D,21} = q_{D,22}$. In the second region, restructuring is immediate if the current state is 2. Therefore, $q_{D,21} = q_{D,22} = 0$. To find $q_{D,11}$ and $q_{D,12}$, we must solve the ode's

$$\frac{1}{2}X^2\sigma_{X,1}^2\frac{d^2q_{D,1j}}{dX^2} + X\hat{\theta}_1\frac{dq_{D,1j}}{dX} - (r_1 + \widehat{\lambda}_1)q_{D,1j} = 0.$$

The general solutions of the above ode are

$$\begin{aligned}
q_{D,11} &= \sum_{m=1}^{2} g_{11,m}X^{l_m},\\
q_{D,12} &= \sum_{m=1}^{2} g_{12,m}X^{l_m}.
\end{aligned}$$

In the third region,

$$\left(\frac{1}{2}\begin{bmatrix}\sigma_{1,X}^2 & 0\\ 0 & \sigma_{2,X}^2\end{bmatrix}X^2\frac{d^2}{dX^2} + \begin{bmatrix}\widehat{\theta}_1 & 0\\ 0 & \widehat{\theta}_2\end{bmatrix}X\frac{d}{dX} - \begin{bmatrix}r_1 & 0\\ 0 & r_2\end{bmatrix} + \begin{bmatrix}-\widehat{\lambda}_1 & \widehat{\lambda}_1\\ \widehat{\lambda}_2 & -\widehat{\lambda}_2\end{bmatrix}\right)\begin{bmatrix}q_{D,11} & q_{D,12}\\ q_{D,21} & q_{D,22}\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}.$$

The above ode system has the following general solution

$$\begin{aligned}
q_{D,1j} &= \sum_{m=1}^{4} h_{1j,m}X^{k_m},\\
q_{D,2j} &= \sum_{m=1}^{4} \epsilon(k_m)h_{1j,m}X^{k_m}.
\end{aligned}$$

In the fourth region default is immediate if the state is 1, but not if the state is 2. Therefore, $q_{D,11} = 1$ and $q_{D,12} = 0$. Also,

$$\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{D,21}}{dX^2} + X\hat{\theta}_2\frac{dq_{D,21}}{dX} + \widehat{\lambda}_2(1 - q_{D,21}) - r_2q_{D,21} = 0,$$

and

$$\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{D,22}}{dX^2} + X\hat{\theta}_2\frac{dq_{D,22}}{dX} + \widehat{\lambda}_2(0 - q_{D,22}) - r_2q_{D,22} = 0.$$

Therefore,

$$\left(\frac{1}{2}\sigma_{X,2}^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\frac{d^2}{dX^2} + \widehat{\theta}_2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\frac{d}{dX} - \left(\widehat{\lambda}_2 + r_2\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} q_{D,21} \\ q_{D,22} \end{bmatrix} + \begin{bmatrix} \widehat{\lambda}_2 \\ 0 \end{bmatrix} = \underline{0}.$$

The general solutions of the above ode's are

$$\begin{aligned} q_{D,21} &= s_{11,1}X^{j_1} + s_{11,2}X^{j_2} + \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2}, \\ q_{D,22} &= s_{12,1}X^{j_1} + s_{12,2}X^{j_2}. \end{aligned}$$

Therefore, in summary we have (S-20). To determine the 16 constants, $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$, we solve 16 simultaneous linear equations, which arise from the 16 boundary conditions:

$$\begin{aligned} &q_{D,11}|_{X\downarrow X_{U,1}} = q_{D,11}|_{X\uparrow X_{U,1}},\ q_{D,21}|_{X\downarrow X_{U,2}} = q_{D,21}|_{X\uparrow X_{U,2}},\ q_{D,11}|_{X\downarrow X_{D,1}} = q_{D,11}|_{X\uparrow X_{D,1}},\ q_{D,21}|_{X\downarrow X_{D,2}} = q_{D,21}|_{X\uparrow X_{D,2}}, \\ &q_{D,11}|_{X\downarrow X_{U,2}} = q_{D,11}|_{X\uparrow X_{U,2}},\ q'_{D,11}|_{X\downarrow X_{U,2}} = q'_{D,11}|_{X\uparrow X_{U,2}},\ q_{D,21}|_{X\downarrow X_{D,1}} = q_{D,21}|_{X\uparrow X_{D,1}},\ q'_{D,21}|_{X\downarrow X_{D,1}} = q'_{D,21}|_{X\uparrow X_{D,1}}, \\ &q_{D,12}|_{X\downarrow X_{U,1}} = q_{D,12}|_{X\uparrow X_{U,1}},\ q_{D,22}|_{X\downarrow X_{U,2}} = q_{D,22}|_{X\uparrow X_{U,2}},\ q_{D,12}|_{X\downarrow X_{D,1}} = q_{D,12}|_{X\uparrow X_{D,1}},\ q_{D,22}|_{X\downarrow X_{D,2}} = q_{D,22}|_{X\uparrow X_{D,2}}, \\ &q_{D,12}|_{X\downarrow X_{U,2}} = q_{D,12}|_{X\uparrow X_{U,2}},\ q'_{D,12}|_{X\downarrow X_{U,2}} = q'_{D,12}|_{X\uparrow X_{U,2}},\ q_{D,22}|_{X\downarrow X_{D,1}} = q_{D,22}|_{X\uparrow X_{D,1}},\ q'_{D,22}|_{X\downarrow X_{D,1}} = q'_{D,22}|_{X\uparrow X_{D,1}}. \end{aligned} \tag{S-24}$$

When $X_{U,1} < X_{U,2}$, we must solve for Arrow-Debreu default and restructuring claims in the following 5 regions:

1. $X > X_{U,2}$
2. $X_{U,1} \le X \le X_{U,2}$
3. $X_{D,1} \le X \le X_{U,1}$
4. $X_{D,2} \le X \le X_{D,1}$
5. $X \le X_{D,2}$

In the first of the above regions, restructuring is immediate whatever the current state is. Therefore, $q_{U,11} = q_{U,22} = 1$ and $q_{U,12} = q_{U,21} = 0$. In the second region, restructuring is immediate when the state is 1, but not when the state is 2. Therefore, $q_{U,11} = 0$, $q_{U,12} = 0$ and

$$\begin{aligned}
\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{U,21}}{dX^2} + X\hat{\theta}_1\frac{dq_{U,21}}{dX} + \widehat{\lambda}_2(1-q_{U,21}) - r_1q_{U,21} &= 0, \\
\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{U,22}}{dX^2} + X\hat{\theta}_1\frac{dq_{U,22}}{dX} + \widehat{\lambda}_2(0-q_{U,22}) - r_1q_{U,22} &= 0.
\end{aligned}$$

Therefore,

$$\left(\frac{1}{2}\sigma_{X,2}^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\frac{d^2}{dX^2} + \widehat{\theta}_2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\frac{d}{dX} - \left(\widehat{\lambda}_2 + r_2\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} q_{U,21} \\ q_{U,22} \end{bmatrix} + \begin{bmatrix} \widehat{\lambda}_2 \\ 0 \end{bmatrix} = \underline{0}.$$

The general solutions of the above ode's are

$$\begin{aligned}
q_{U,21} &= g_{11,1}X^{j_1} + g_{11,2}X^{j_2} + \frac{\widehat{\lambda}_2}{r_2+\widehat{\lambda}_2}, \\
q_{U,22} &= g_{12,1}X^{j_1} + g_{12,2}X^{j_2}.
\end{aligned}$$

In the third region

$$\left(\frac{1}{2}\begin{bmatrix} \sigma_{1,X}^2 & 0 \\ 0 & \sigma_{2,X}^2 \end{bmatrix}X^2\frac{d^2}{dX^2} + \begin{bmatrix} \widehat{\theta}_1 & 0 \\ 0 & \widehat{\theta}_2 \end{bmatrix}X\frac{d}{dX} - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} + \begin{bmatrix} -\widehat{\lambda}_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 \end{bmatrix}\right)\begin{bmatrix} q_{U,11} & q_{U,12} \\ q_{U,21} & q_{U,22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The above ode system has the following general solution

$$\begin{aligned}
q_{U,1j} &= \sum_{m=1}^{4} h_{1j,m}X^{k_m}, \\
q_{U,2j} &= \sum_{m=1}^{4} \epsilon(k_m)h_{1j,m}X^{k_m}.
\end{aligned}$$

In the fourth region default is immediate if the state is 1, but not if the state is 2. Therefore, $q_{U,11} = q_{U,12} = 0$ and

$$\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{U,2j}}{dX^2} + X\hat{\theta}_2\frac{dq_{U,2j}}{dX} - (r_2+\widehat{\lambda}_2)q_{U,2j} = 0.$$

The general solution of the above ode is

$$q_{U,2j} = \sum_{m=1}^{2} s_{2j,m}X^{j_m}.$$

In summary, we have (S-21). To determine the 16 constants, $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$, we solve 16 simultaneous linear equations, which arise from the

16 boundary conditions:

$$
\begin{array}{c}
q_{U,11}|_{X\downarrow X_{U,1}} = q_{U,11}|_{X\uparrow X_{U,1}},\ q_{U,11}|_{X\downarrow X_{D,1}} = q_{U,11}|_{X\uparrow X_{D,1}},\ q_{U,21}|_{X\downarrow X_{U,2}} = q_{U,21}|_{X\uparrow X_{U,2}},\ q_{U,21}|_{X\downarrow X_{U,1}} = q_{U,21}|_{X\uparrow X_{U,1}},\\
q_{U,21}|_{X\downarrow X_{D,1}} = q_{U,21}|_{X\uparrow X_{D,1}},\ q_{U,21}|_{X\downarrow X_{D,2}} = q_{U,21}|_{X\uparrow X_{D,2}},\ q'_{U,21}|_{X\downarrow X_{U,1}} = q'_{U,21}|_{X\uparrow X_{U,1}},\ q'_{U,21}|_{X\downarrow X_{D,1}} = q'_{U,21}|_{X\uparrow X_{D,1}},\\
q_{U,12}|_{X\downarrow X_{U,1}} = q_{U,12}|_{X\uparrow X_{U,1}},\ q_{U,12}|_{X\downarrow X_{D,1}} = q_{U,12}|_{X\uparrow X_{D,1}},\ q_{U,22}|_{X\downarrow X_{U,2}} = q_{U,22}|_{X\uparrow X_{U,2}},\ q_{U,22}|_{X\downarrow X_{U,1}} = q_{U,22}|_{X\uparrow X_{U,1}},\\
q_{U,22}|_{X\downarrow X_{D,1}} = q_{U,22}|_{X\uparrow X_{D,1}},\ q_{U,22}|_{X\downarrow X_{D,2}} = q_{U,22}|_{X\uparrow X_{D,2}},\ q'_{U,22}|_{X\downarrow X_{U,1}} = q'_{U,22}|_{X\uparrow X_{U,1}},\ q'_{U,22}|_{X\downarrow X_{D,1}} = q'_{U,22}|_{X\uparrow X_{D,1}}.
\end{array}
\tag{S-25}
$$

We now solve for Arrow-Debreu default claims. In the first region, restructuring is immediate whatever the current state is. Therefore, $q_{D,11} = q_{D,12} = q_{D,21} = q_{D,22} = 0$. In the second region, restructuring is immediate when the state is 1, but not when the state is 2. Therefore, $q_{D,11} = q_{D,12} = 0$ and

$$
\begin{aligned}
\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{D,21}}{dX^2} + X\hat{\theta}_1\frac{dq_{D,21}}{dX} + \widehat{\lambda}_2(0 - q_{D,21}) - r_1q_{D,21} &= 0,\\
\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{D,22}}{dX^2} + X\hat{\theta}_1\frac{dq_{D,22}}{dX} + \widehat{\lambda}_2(0 - q_{D,22}) - r_1q_{D,22} &= 0.
\end{aligned}
$$

Therefore,

$$
\left(\frac{1}{2}\sigma_{X,2}^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\frac{d^2}{dX^2} + \widehat{\theta}_2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\frac{d}{dX} - \left(\widehat{\lambda}_2 + r_2\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\begin{bmatrix}q_{D,21}\\ q_{D,22}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}.
$$

The general solutions of the above ode's are

$$
\begin{aligned}
q_{D,21} &= g_{11,1}X^{j_1} + g_{11,2}X^{j_2},\\
q_{D,22} &= g_{12,1}X^{j_1} + g_{12,2}X^{j_2}.
\end{aligned}
$$

In the third region

$$
\left(\frac{1}{2}\begin{bmatrix}\sigma_{1,X}^2 & 0\\ 0 & \sigma_{2,X}^2\end{bmatrix}X^2\frac{d^2}{dX^2} + \begin{bmatrix}\widehat{\theta}_1 & 0\\ 0 & \widehat{\theta}_2\end{bmatrix}X\frac{d}{dX} - \begin{bmatrix}r_1 & 0\\ 0 & r_2\end{bmatrix} + \begin{bmatrix}-\widehat{\lambda}_1 & \widehat{\lambda}_1\\ \widehat{\lambda}_2 & -\widehat{\lambda}_2\end{bmatrix}\right)\begin{bmatrix}q_{D,11} & q_{D,12}\\ q_{D,21} & q_{D,22}\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}.
$$

The above ode system has the following general solution

$$
\begin{aligned}
q_{D,1j} &= \sum_{m=1}^{4} h_{1j,m}X^{k_m},\\
q_{D,2j} &= \sum_{m=1}^{4} \epsilon(k_m)h_{1j,m}X^{k_m}.
\end{aligned}
$$

In the fourth region default is immediate if the state is 1, but not if the state is 2. Therefore, $q_{D,11} = 1$ and $q_{D,12} = 0$, and so

$$
\begin{aligned}
\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{D,21}}{dX^2} + X\hat{\theta}_2\frac{dq_{D,21}}{dX} + \widehat{\lambda}_2(1 - q_{D,21}) - r_2q_{D,21} &= 0,\\
\frac{1}{2}X^2\sigma_{X,2}^2\frac{d^2q_{D,22}}{dX^2} + X\hat{\theta}_2\frac{dq_{D,22}}{dX} + \widehat{\lambda}_2(0 - q_{D,22}) - r_2q_{D,22} &= 0.
\end{aligned}
$$

The general solutions of the above ode's are

$$\begin{aligned} q_{D,21} &= \sum_{m=1}^{2} s_{22,m} X^{j_m} + \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2}, \\ q_{D,22} &= \sum_{m=1}^{2} s_{22,m} X^{j_m}. \end{aligned}$$

In summary, we have (S-22). To determine the 16 constants, $g_{11,1}$, $g_{11,2}$, $h_{11,1}$, $h_{11,2}$, $h_{11,3}$, $h_{11,4}$, $g_{12,1}$, $g_{12,2}$, $h_{12,1}$, $h_{12,2}$, $h_{12,3}$, $h_{12,4}$, $s_{11,1}$, $s_{11,2}$, $s_{12,1}$, $s_{12,2}$, we solve 16 simultaneous linear equations, which arise from the 16 boundary conditions:

$$\begin{gathered} q_{D,11}|_{X\downarrow X_{U,1}} = q_{D,11}|_{X\uparrow X_{U,1}}, \; q_{D,11}|_{X\downarrow X_{D,1}} = q_{D,11}|_{X\uparrow X_{D,1}}, \; q_{D,21}|_{X\downarrow X_{U,2}} = q_{D,21}|_{X\uparrow X_{U,2}}, \; q_{D,21}|_{X\downarrow X_{U,1}} = q_{D,21}|_{X\uparrow X_{U,1}}, \\ q_{D,21}|_{X\downarrow X_{D,1}} = q_{D,21}|_{X\uparrow X_{D,1}}, \; q_{D,21}|_{X\downarrow X_{D,2}} = q_{D,21}|_{X\uparrow X_{D,2}}, \; q'_{D,21}|_{X\downarrow X_{U,1}} = q'_{D,21}|_{X\uparrow X_{U,1}}, \; q'_{D,21}|_{X\downarrow X_{D,1}} = q'_{D,21}|_{X\uparrow X_{D,1}}, \\ q_{D,12}|_{X\downarrow X_{U,1}} = q_{D,12}|_{X\uparrow X_{U,1}}, \; q_{D,12}|_{X\downarrow X_{D,1}} = q_{D,12}|_{X\uparrow X_{D,1}}, \; q_{D,22}|_{X\downarrow X_{U,2}} = q_{D,22}|_{X\uparrow X_{U,2}}, \; q_{D,22}|_{X\downarrow X_{U,1}} = q_{D,22}|_{X\uparrow X_{U,1}}, \\ q_{D,22}|_{X\downarrow X_{D,1}} = q_{D,22}|_{X\uparrow X_{D,1}}, \; q_{D,22}|_{X\downarrow X_{D,2}} = q_{D,22}|_{X\uparrow X_{D,2}}, \; q'_{D,22}|_{X\downarrow X_{U,1}} = q'_{D,22}|_{X\uparrow X_{U,1}}, \; q'_{D,22}|_{X\downarrow X_{D,1}} = q'_{D,22}|_{X\uparrow X_{D,1}}. \end{gathered} \tag{S-26}$$

■

## S-IV Further details on the risk adjustment

In this section, we give further details on the comparative statics of the risk adjustment and details of how it is computed.

Comparative statics of the risk-adjustment are economically intuitive. Since more intertemporal risk increases the risk price associated with shifting into the bad state, $\mathcal{R}_{\nu_t 1}$ increases and $\mathcal{R}_{\nu_t 2}$ decreases as the half-life of the Markov chain governing changes in the state increases. When an economic downturn is more severe, the risk price associated with shifting into the bad state is higher. Therefore, an increase in consumption growth volatility and a decrease in expected consumption growth have a similar effect on the risk-adjustment. As the risk-neutral expected earnings growth rate decreases (e.g. because the expected earnings growth decreases or systematic earnings growth volatility increases), the risk of defaulting in the bad state increases, thus lowering $\mathcal{R}_{\nu_t 1}$.

Interestingly, the effect of firm-specific factors is quite different from the effect of consumption growth. For example, increasing idiosyncratic earnings growth volatility makes both good and bad states worse, but decreases all risk-adjustment factors, since there is less risk associated with whether default will take place or not within a given time interval. Similarly, higher default boundaries lead to a fall in risk-adjustment factors. As we show in the paper, higher leverage leads to optimally higher default boundaries, and thus risk-adjustment factors are decreasing with respect to leverage as well.

We estimate the risk adjustments reported in Table III of the paper as follows. Along with default probabilities, we compute the risk-adjustment, $\mathcal{R}_{\nu_t \nu_D} = \hat{p}_{D,\nu_t \nu_D}/p_{D,\nu_t \nu_D}$, for individual firms and then for each firm we compute $\mathcal{R}_i = f_1 \mathcal{R}_{i1} + \mathcal{R}_{i2}$ and hence $\mathcal{R} = f_1 \mathcal{R}_1 + f_2 \mathcal{R}_2$, which is reported in Panels A and B. Then we compute this quantity under cross-sectional dynamics, which is reported in Panel C. Thus, due to averaging over states, the risk-adjustments reported in Table do not satisfy the relationship, $q_D = \mathcal{T}\mathcal{R}p_D$. Note also, that the $\mathcal{R}_{\nu_t\nu_D} = \hat{p}_{D,\nu_t\nu_D}/p_{D,\nu_t\nu_D}$ implies that the risk adjustment $\mathcal{R}_{\nu_t\nu_D}$ is very sensitive to the value of the the actual probability, $p_{D,\nu_t\nu_D}$, and one needs only small changes in this number for our model implied risk adjustments to be much closer to those in Berndt et al. (2005).

# S-V Finite maturity corporate bonds

**Proposition S-3** *Conditional on the current state being $i$, the date-$t$ price of a zero coupon risk-free bond which pays out one unit of consumption at date $T$, is*

$$B^0_{f,i,T-t} = E_t\left[\frac{\pi_T}{\pi_t}|\nu_t = i\right] = [e^{-(diag(r_1,r_2)-\hat{\Lambda})(T-t)}(1,1)^T]_i, \tag{S-27}$$

*where $diag(r_1,r_2)$ is the 2 by 2 diagonal matrix with $r_1$ and $r_2$ along the diagonal and $\hat{\Lambda} - \begin{bmatrix} -\widehat{\lambda}_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 \end{bmatrix}$ is the risk-neutral generator matrix.*

*Conditional on the current state being $i$, the date-$t$ price of a zero coupon risk-free bond which pays out one unit of consumption at date $T$, if the economy is in state $j$ at date $T$ is*

$$B^0_{f,i,j,T-t} = E_t\left[\frac{\pi_T}{\pi_t}\Pr(\nu_T = j|\nu_t = i)|\nu_t = i\right] = \widehat{\Pr}(\nu_T = j|\nu_t = i)B^0_{f,i,T-t}. \tag{S-28}$$

**Proof of Proposition S-3.** In this proof it is not necessary to distinguish between the state of the economy at dates $t-$ and $t$. By definition

$$B^0_{f,i,T-t} = E_t\left[\frac{\pi_T}{\pi_t}|\nu_t = i\right].$$

The basic asset pricing equation implies that

$$\frac{\partial B^0_{f,i,T-t}}{\partial t} + \sum_{j\neq i}\widehat{\lambda}_{ij}(B^0_{f,j,T-t} - B^0_{f,i,T-t}) - r_i B^0_{f,i,T-t} = 0,$$

The above equation can be rewritten in matrix form as

$$\frac{\partial \underline{B}^0_{f,T-t}}{\partial t} + [\widehat{\Lambda} - diag(r_1,r_2)]\underline{B}^0_{f,T-t} = \underline{0}_2,$$

where $\underline{B}^0_{f,T-t} = (B^0_{f,1,T-t}, B^0_{f,2,T-t})^T$, $\underline{0}_2 = (0,0)^T$, $\widehat{\Lambda}$ is the risk-neutral generator matrix and $diag(r_1,r_2)$ is the 2 by 2 diagonal matrix with $r_1$ and $r_2$ along the diagonal. To solve the above differential equation system, define $\underline{y}$ via

$$\underline{B}^0_{f,T-t} = E\underline{y},$$

where $E = (\underline{e}_1, \underline{e}_2)$ is the matrix of column eigenvectors of $\widehat{\Lambda} - diag(r_1,r_2)$, i.e.

$$[\widehat{\Lambda} - diag(r_1,r_2)]\underline{e}_i = \omega_i\underline{e}_i,\ i \in \{1,2\}.$$

Then

$$\begin{aligned} E\frac{\partial \underline{y}}{\partial t} + [\widehat{\Lambda} - diag(r_1, r_2)]E\underline{y} &= \underline{0}_2 \\ \frac{\partial \underline{y}}{\partial t} + E^{-1}[\widehat{\Lambda} - diag(r_1, r_2)]E\underline{y} &= \underline{0}_2 \\ \frac{\partial \underline{y}}{\partial t} + D\underline{y} &= \underline{0}_2, \end{aligned}$$

where $D = E^{-1}[\widehat{\Lambda} - diag(r_1, r_2)]E$ is a diagonal matrix. Solving the above differential equation system gives

$$\underline{y} = e^{-Dt}\underline{m},$$

where $\underline{m}$ is a 2 by 1 vector of constants of integration. Hence,

$$\begin{aligned} \underline{B}^0_{f,T-t} &= Ee^{-Dt}\underline{m}, \\ &= Ee^{-Dt}E^{-1}\underline{b}, \end{aligned}$$

where $\underline{b}$ is a 2 by 1 vector of constants of integration. Since $\underline{B}^0_{f,0} = (1,1)^T$, it follows that

$$\begin{aligned} \underline{B}^0_{f,T-t} &= Ee^{-Dt}E^{-1}(1,1)^T, \\ &= e^{-(diag(r_1,r_2)-\widehat{\Lambda})(T-t)}. \end{aligned}$$

Equation (S-27) follows.

The price of a zero coupon risk-free bond which pays out one unit of consumption at date $T$, if the economy is in state $j$ at date $T$ is just $B^0_{f,i,T-t}$ multiplied by the risk-neutral probability of being in state $j$ at date $T$, conditional on being in state $i$ at date $t$. Equation (S-28) follows. ■

**Proposition S-4** *Finite maturity risk-free debt pays a coupon at the rate $c$ until maturity (time $T$) and the amount $P$ at maturity. The time-$t$ price of finite maturity risk-free debt when the current state is $i$ is given by*

$$B_{f,i,T-t} = c\left(\frac{1}{r_{P,i}} - \sum_{i=1}^{2}\frac{B^0_{f,i,j,T-t}}{r_{P,j}}\right) + PB^0_{i,t,T}. \tag{S-29}$$

**Proof of Proposition S-4.** The date-$t$ value of finite maturity risk-free debt is

$$B_{f,i,T-t} = cE_t\left[\int_t^T \frac{\pi_s}{\pi_t}ds|\nu_t = i\right] + PE_t\left[\frac{\pi_T}{\pi_t}|\nu_t = i\right],$$

conditional on the current state being $i$. Hence,

$$B_{f,i,T-t} = cE_t\left[\int_t^\infty \frac{\pi_s}{\pi_t}ds|\nu_t = i\right] - E_t\left[\int_T^\infty \frac{\pi_s}{\pi_t}ds|\nu_t = i\right] + PE_t\left[\frac{\pi_T}{\pi_t}|\nu_t = i\right].$$

We know that

$$E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t}ds|\nu_t = i\right] = \frac{1}{r_{P,i}}.$$

Furthermore,

$$
\begin{aligned}
E_t\left[\int_T^{\infty}\frac{\pi_s}{\pi_t}ds|\nu_t=i\right] &= E_t\left[\frac{\pi_T}{\pi_t}|\nu_t=i\right]\sum_{j=1}^{2}E_T\left[\int_T^{\infty}\frac{\pi_s}{\pi_T}ds|\nu_T=j\right]\hat{\Pr}(\nu_T=j|\nu_t=i)\\
&= \sum_{j=1}^{2}E_T\left[\int_T^{\infty}\frac{\pi_s}{\pi_T}ds|\nu_T=j\right]\hat{\Pr}(\nu_T=j|\nu_t=i)E_t\left[\frac{\pi_T}{\pi_t}|\nu_t=i\right]\\
&= \sum_{j=1}^{2}\frac{1}{r_{P,j}}B^0_{f,i,j,T-t},
\end{aligned}
$$

and

$$
E_t\left[\frac{\pi_T}{\pi_t}|\nu_t=i\right]=B^0_{i,T-t}.
$$

Equation (S-29) follows. ■

**Proposition S-5** *Finite maturity corporate debt pays a coupon at the rate $c$ until default (the random time, $\tau_D$) or maturity (time $T$), whichever is earlier, and the amount $P$ at maturity, if default has not already occurred. The time-$t$ price of finite maturity corporate debt is*

$$
\begin{aligned}
B_{i,T-t} &= c\left(\frac{1}{r_{P,i}}-\sum_{j=1}^{2}\frac{1}{r_{P,j}}B^0_{f,ij,T-t}(1-\widehat{p}_{D,i,T-t})-\sum_{j=1}^{2}\frac{1}{r_{P,j}}q_{D,ij,T-t}\right)+PB^0_{f,i,T-t}(1-\widehat{p}_{D,i,T-t})\\
&\quad+\sum_{j=1}^{2}q_{D,ij,T-t}\alpha_jA_j(X_{D,j}), \qquad \text{(S-30)}
\end{aligned}
$$

*where $\hat{p}_{D,i,T-t}$ is the risk-neutral probability of default occurring before time $T$, conditional on the current state being $i$ and $q_{D,ij,T-t}$ is the price of an Arrow-Debreu security, which pays of a unit of consumption at default, if default occurs in state $j$ and before time $T$, conditional on the current state being $i$.*

*Closed-form expressions for $B^0_{f,ij,T-t}$ and $B^0_{f,i,T-t}$ are given in the proposition above, whereas $\hat{p}_{D,i,T-t}$ and $q_{D,ij,T-t}$ are computed by Monte-Carlo simulation.*

**Proof of Proposition S-5.** The date-$t$ value of finite maturity corporate debt is

$$
B_{i,T-t}=cE_t\left[\int_t^{\min(\tau_D,T)}\frac{\pi_s}{\pi_t}ds|\nu_t=i\right]+PE_t\left[\frac{\pi_T}{\pi_t}1_{\{\tau_D>T\}}ds|\nu_t=i\right]+E_t\left[\frac{\pi_{\tau_D}}{\pi_t}1_{\{\tau_D\le T\}}A_{\nu_{\tau_D}}(X_{\tau_D})ds|\nu_t=i\right].
$$

We simplify the above expression term by term.

$$
E_t\left[\int_t^{\min(\tau_D,T)}\frac{\pi_s}{\pi_t}ds|\nu_t=i\right] = E_t\left[\int_t^{T}\frac{\pi_s}{\pi_t}1_{\{\tau_D>T\}}ds|\nu_t=i\right]+E_t\left[\int_t^{\tau_D}\frac{\pi_s}{\pi_t}1_{\{\tau_D\le T\}}ds|\nu_t=i\right].
$$

The first term in the above expression can be rewritten as

$$\begin{aligned}
E_t\left[\int_t^T \frac{\pi_s}{\pi_t} 1_{\{\tau_D > T\}} ds | \nu_t = i\right] &= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D > T\}} ds | \nu_t = i\right] - E_t\left[\int_T^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D > T\}} ds | \nu_t = i\right] \\
&= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D > T\}} ds | \nu_t = i\right] \\
&\quad - E_t\left[\frac{\pi_T}{\pi_t} 1_{\{\tau_D > T\}} | \nu_t = i\right] \sum_{j=1}^{2} E_T\left[\int_T^\infty \frac{\pi_s}{\pi_T} ds | \nu_T = j\right] \widehat{\Pr}(\nu_T = j | \nu_t = i) \\
&= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D > T\}} ds | \nu_t = i\right] \\
&\quad - \sum_{j=1}^{2} E_T\left[\int_T^\infty \frac{\pi_s}{\pi_T} ds | \nu_T = j\right] E_t\left[\frac{\pi_T}{\pi_t} 1_{\{\tau_D > T\}} \widehat{\Pr}(\nu_T = j | \nu_t = i)\right] \\
&= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D > T\}} ds | \nu_t = i\right] - \sum_{j=1}^{2} \frac{1}{r_{P,j}} E_t\left[\frac{\pi_T}{\pi_t} 1_{\{\tau_D > T\}} \widehat{\Pr}(\nu_T = j | \nu_t = i)\right] \\
&= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D > T\}} ds | \nu_t = i\right] - \sum_{j=1}^{2} \frac{1}{r_{P,j}} B^0_{f,i,j,T-t}(1 - \widehat{p}_{D,i,T-t}).
\end{aligned}$$

Similarly,

$$\begin{aligned}
E_t\left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} 1_{\{\tau_D \le T\}} ds | \nu_t = i\right] &= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D \le T\}} ds | \nu_t = i\right] - E_t\left[\int_{\tau_D}^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D \le T\}} ds | \nu_t = i\right] \\
&= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D \le T\}} ds | \nu_t = i\right] \\
&\quad - E_t\left[\frac{\pi_{\tau_D}}{\pi_t} 1_{\{\tau_D \le T\}} | \nu_t = i\right] \sum_{j=1}^{2} E_{\tau_D}\left[\int_{\tau_D}^\infty \frac{\pi_s}{\pi_{\tau_D}} ds | \nu_{\tau_D} = j\right] \widehat{\Pr}(\nu_{\tau_D} = j | \nu_t = i) \\
&= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D \le T\}} ds | \nu_t = i\right] \\
&\quad - \sum_{j=1}^{2} E_{\tau_D}\left[\int_{\tau_D}^\infty \frac{\pi_s}{\pi_{\tau_D}} ds | \nu_{\tau_D} = j\right] E_t\left[\frac{\pi_{\tau_D}}{\pi_t} 1_{\{\tau_D \le T\}} \widehat{\Pr}(\nu_{\tau_D} = j | \nu_t = i) | \nu_t = i\right] \\
&= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} 1_{\{\tau_D \le T\}} ds | \nu_t = i\right] - \sum_{j=1}^{2} \frac{1}{r_{P,j}} q_{D,ij,T-t}\,.
\end{aligned}$$

Therefore,

$$\begin{aligned}
E_t\left[\int_t^{\min(\tau_D, T)} \frac{\pi_s}{\pi_t} ds | \nu_t = i\right] &= E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} ds | \nu_t = i\right] - \sum_{j=1}^{2} \frac{1}{r_{P,j}} B^0_{f,i,j,T-t}(1 - \widehat{p}_{D,i,T-t}) - \sum_{j=1}^{2} \frac{1}{r_{P,j}} q_{D,ij,T-t} \\
&= \frac{1}{r_{P,i}} - \sum_{j=1}^{2} \frac{1}{r_{P,j}} B^0_{f,i,j,T-t}(1 - \widehat{p}_{D,i,T-t}) - \sum_{j=1}^{2} \frac{1}{r_{P,j}} q_{D,ij,T-t}.
\end{aligned}$$

Now we simplify $E_t\left[\frac{\pi_T}{\pi_t}1_{\{\tau_D>T\}}ds|\nu_t=i\right]$:

$$\begin{aligned}E_t\left[\frac{\pi_T}{\pi_t}1_{\{\tau_D>T\}}ds|\nu_t=i\right] &= E_t\left[\frac{\pi_T}{\pi_t}ds|\nu_t=i\right]-E_t\left[\frac{\pi_T}{\pi_t}1_{\{\tau_D\leq T\}}ds|\nu_t=i\right]\\ &= E_t\left[\frac{\pi_T}{\pi_t}ds|\nu_t=i\right]-E_t\left[\frac{\pi_T}{\pi_t}ds|\nu_t=i\right]\widehat{p}_{D,i,T-t}\\ &= B^0_{f,i,T-t}(1-\widehat{p}_{D,i,T-t}).\end{aligned}$$

Simplifying $E_t\left[\frac{\pi_{\tau_D}}{\pi_t}1_{\{\tau_D\leq T\}}A_{\nu_{\tau_D}}(X_{\tau_D})ds|\nu_t=i\right]$ gives

$$\begin{aligned}E_t\left[\frac{\pi_{\tau_D}}{\pi_t}1_{\{\tau_D\leq T\}}A_{\nu_{\tau_D}}(X_{\tau_D})ds|\nu_t=i\right] &= \sum_{j=1}^{2}E_t\left[\frac{\pi_{\tau_D}}{\pi_t}1_{\{\tau_D\leq T\}}A_{\nu_{\tau_D}}(X_{\tau_D})ds|\nu_t=i,\nu_{\tau_D=j}\Pr(\nu_{\tau_D}=j|\nu_t=i)\right]\\ &= \sum_{j=1}^{2}E_t\left[\frac{\pi_{\tau_D}}{\pi_t}1_{\{\tau_D\leq T\}}A_j(X_{D,j})ds\Pr(\nu_{\tau_D}=j|\nu_t=i)|\nu_t=i,\nu_{\tau_D=j}\right]\\ &= \sum_{j=1}^{2}E_t\left[\frac{\pi_{\tau_D}}{\pi_t}1_{\{\tau_D\leq T\}}ds\Pr(\nu_{\tau_D}=j|\nu_t=i)|\nu_t=i,\nu_{\tau_D=j}\right]A_j(X_{D,j})\\ &= \sum_{j=1}^{2}q_{D,ij,T-t}\alpha_jA_j(X_{D,j}).\end{aligned}$$

Hence,

$$\begin{aligned}B_{i,T-t} &= cE_t\left[\int_t^{\min(\tau_D,T)}\frac{\pi_s}{\pi_t}ds|\nu_t=i\right]+PE_t\left[\frac{\pi_T}{\pi_t}1_{\{\tau_D>T\}}ds|\nu_t=i\right]+E_t\left[\frac{\pi_{\tau_D}}{\pi_t}1_{\{\tau_D\leq T\}}A_{\nu_{\tau_D}}(X_{\tau_D})ds|\nu_t=i\right]\\ &= c\left(\frac{1}{r_{P,i}}-\sum_{j=1}^{2}\frac{1}{r_{P,j}}B^0_{f,i,j,T-t}(1-\widehat{p}_{D,i,T-t})-\sum_{j=1}^{2}\frac{1}{r_{P,j}}q_{D,ij,T-t}\right)\\ &\quad +PB^0_{f,i,T-t}(1-\widehat{p}_{D,i,T-t})+\sum_{j=1}^{2}q_{D,ij,T-t}\alpha_jA_j(X_{D,j}).\end{aligned}$$

■

# S-VI Estimation details

To estimate a Markov switching model as described in Hamilton (1989), we start by assuming that aggregate consumption, $C$, is given by

$$c_{t+1}=\Delta\log C_{t+1}=g_i-\frac{1}{2}\sigma^2_{C,i}+\sigma_{C,i}\varepsilon_{C,t+1}, \tag{S-31}$$

and aggregate earnings, $X=\sum_{n=1}^{N}X_n$, by

$$x_{t+1}=\Delta\log X_{t+1}=\theta_i-\frac{1}{2}(\sigma^s_{X,i})^2+\sigma^s_{X,i}\varepsilon_{X,t+1}, \tag{S-32}$$

where shocks to earnings growth and consumption growth are normally distributed with zero mean and unit variance and correlation of $\rho_{XC}$. The joint normal distribution for earnings growth and consumption growth denoted by $\Phi$

$$\Phi(x_t, c_t|\nu_t = i, \Omega_{t-1}; \Gamma) = \frac{1}{2\pi\sigma^s_{X,i}\sigma_{C,i}\sqrt{1-\rho^2_{XC}}} \exp\{-1/2(\varepsilon^2_{X,t} + \varepsilon^2_{C,t} - 2\rho_{XC}\varepsilon_{X,t}\varepsilon_{C,t})/(1-\rho^2_{XC})\}.$$

where $\Omega_t$ denotes the set of all observations up to time $t$ and $\Gamma$ the set of unknown parameters. Having obtained our parameter estimates by maximizing the log-likelihood, we must obtain the parameters of the continuous-time Markov chain from the estimated discrete-time transition matrix

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix},$$

where $P_{ij} = P(\nu_{t+1} = \theta_j|\nu_t = \theta_i)$ is the probability of switching from state $i$ to $j$ within a quarter. To do this note that the matrix of quarterly transition probabilities, $P$, is related to the generator of the continuous-time chain

$$\Lambda = \begin{pmatrix} -\lambda_1 & \lambda_1 \\ \lambda_2 & -\lambda_2 \end{pmatrix}$$

by $e^{\Lambda\frac{1}{4}} = P$. Using standard techniques from linear algebra, we can show that

$$e^{\Lambda\frac{1}{4}} = \begin{pmatrix} f_1 & f_2 \\ f_1 & f_2 \end{pmatrix} + \begin{pmatrix} f_2 & -f_2 \\ -f_1 & f_1 \end{pmatrix} e^{-\frac{1}{4}p}, \tag{S-33}$$

Where $p = \lambda_1 + \lambda_2$ and $f_i = \frac{\lambda_j}{p}$, $i, j \in \{1, 2\}$, $j \neq i$. Equating (S-33) with $P$ implies that

$$f_1 = \left(1 + \frac{P_{12}}{P_{21}}\right)^{-1} \text{ and } p = -4\ln\left(1 - \frac{P_{12}}{1-f_1}\right).$$

# S-VII Comparison with Bansal and Yaron (2004)

The consumption-based asset pricing model we use as a building block for our structural-equilibrium model uses a representative agent with Epstein-Zin-Weil preferences and assumes that the first and second moments of consumption growth are stochastic. The framework we use to model the stochastic behavior of these moments is clearly similar to Bansal and Yaron (2004), but there are also important differences, which we now describe. In particular, we focus on explaining why our model, as calibrated, is *not* a long-run risk model.

Bansal and Yaron (2004) use a discrete-time model, where conditional expected consumption growth and the volatility of consumption growth follow AR(1) processes, such as

$$x_{t+1} = \varrho x_t + \sigma\epsilon_{t+1}, \tag{S-34}$$

where $\epsilon_{t+1}$ follows the standard normal distribution and is independent over time. The parameter $\varrho$ is the *persistence* of the AR(1) process. It follows from (S-34) that

$$E_t[x_{t+s}] = \varrho^s x_t.$$

Therefore, if $|\varrho| < 1$, then the AR(1) process has a long-run (also known as ergodic or stationary) mean of zero, i.e.

$$E[x_t] = \lim_{s\to\infty} E_t[x_{t+s}] = 0.$$

The process converges to its long-run distribution geometrically with rate $\varrho$. Its current distance from its long-run mean is $x_t$ and the mean of the distance from the long-run mean at time $t+s$ is $E_t[x_{t+s}]$. The time it takes for the mean distance from the long-run mean to halve gives the *half-life*, $t_{1/2}$, of the AR(1) process, i.e. $E_t[x_{t+t_{1/2}}] = \frac{1}{2}x_t$, which implies that

$$t_{1/2} = \frac{\ln\frac{1}{2}}{\ln\varrho}.$$

In Bansal and Yaron (2004), the monthly persistence parameter for conditional expected consumption growth is $\varrho = 0.979$, implying a half-life of 32.6592 in months, which is 2.72 years. The longer this half-life is, the slower the convergence of the AR(1) process to its long-run distribution. When $\varrho$ is close to 1 the AR(1) process is difficult to distinguish statistically from a unit root process and convergence to the long-run is slow, resulting in a longer half-life. Models where this is this case, such as Bansal and Yaron (2004), are known as *long-run risk* models. The Markov chain in our framework has a smaller half-life of $t_{1/2} = 0.9065$ years, which, in an AR(1) process, would imply a monthly persistence parameter of 0.4655. This number is clearly much smaller than 1 and results in an AR(1) process, which can be statistically distinguished from a unit root. Furthermore, its convergence to its long-run distribution is much faster than in Bansal and Yaron (2004). Thus, it would be inaccurate to state that our model, as calibrated, is a long-run risk model. In fact, since the Markov chain changes state at a frequency similar to that at which business cycles change, it would be more accurate to describe our model as a *business-cycle risk* model.

The other difference between our framework and Bansal and Yaron (2004) is that, in our model, states are discrete rather than continuous. Furthermore, as we explain in the paper, the average duration of each state can be different, which is impossible with an AR(1) process.

## S-VIII Stripping down the model

In this section, to make it easier to see which assumption in our model is responsible for what effect, we strip down the model by removing intertemporal macroeconomic risk in the first and second moments of earnings and consumption growth. This leaves us with Model 1, where aggregate consumption and earnings growth are i.i.d. and the representative agent has Epstein-Zin-Weil preferences. We then rebuild the model piece-by-piece. In Model 2, we introduce Markov switching in the first and second moments of earnings growth, but not to consumption growth. And finally, in Model 3, we rebuild the model fully by having Markov switching in the first and second moments of consumption growth. To get a fair comparison across models, we calibrate them all to the same data on earnings and consumption and use an optimal default boundary with optimal capital structure. For each model, we produce credit spreads and the levered equity risk premium for an individual firm at it's refinancing point *and* for a dynamic cross-section. Table S-I below summarizes our results.

Note that in Model 1, we use an EIS of 0.75 to ensure that the (before-tax) price/earnings ratio, given by

$$\frac{1}{\overline{r} + \gamma\rho_{XC}\sigma_X\sigma_C - \mu_X}, \tag{S-35}$$

where $\overline{r} = \beta + \frac{1}{\psi} g - \frac{1}{2}\gamma \left(1 + \frac{1}{\psi}\right) \sigma_C^2$, is finite. We can see from the above expressions that if the EIS, $\psi$, is high, (which makes the agent is more tolerant of non smooth intertemporal consumption profiles), then the agent invests more in the risk-free bond, which reduces the locally risk-free rate, $\overline{r}$ and thus increases the price/earnings ratio. However, we must ensure that the EIS is small enough that $\overline{r} + \gamma\rho_{XC}\sigma_X\sigma_C - \mu_X > 0$.

In order to get a fair comparison across models in Table S-I, we must thus set the EIS equal to 0.75 in **all** models (even though in Models 2 and 3, price/earnings ratios are still finite when the EIS is greater than 0.75).

Note that to obtain a high credit spread while keeping actual default probabilities realistically low, our model must produce both a high time adjustment and a high risk adjustment factor. Observe that the time adjustment factor is already quite high in Model 1, and does not increase significantly across models. The reason is that solely by using Epstein–Zin-Weil preferences, we can separate relative risk aversion and EIS and thus obtain a reasonably low risk-free rate (this point was made in Weil (1989)). This implies a high time adjustment factor, without having to introduce Markov switching in the growth rates of macroeconomic fundamentals.

Adding Markov switching to the first and second moments of earnings growth, but not consumption growth (moving from Model 1 to Model 2) does not impact the size of the credit spread and the levered equity risk premium much at all, because these switches are not correlated with the state-price density, and are hence not priced. This is summarized by the fact that the risk-distortion factor, $\omega$, is 1.

Introducing switching in the moments of consumption growth (moving from Model 2 to Model 3) leads to a significant increase in the spread and the risk premium. Switches in the moments of earnings growth are now correlated with the state-price density ($\omega > 1$, so the state-price density jumps up whenever expected earnings growth/earnings growth volatility jumps down/up). Thus switches in earnings growth rates are now priced into credit spreads and the risk premium. This is reflected by an increase in the risk-adjustment factor.

In summary, two assumptions lie behind the model's ability to generate high prices for Arrow-Debreu default claims, without increasing actual default probabilities. The first is the use of Epstein-Zin-Weil preferences, which increases the time-adjustment factor by lowering the risk-free rate. The second is the assumption of switching in the first and second moments of both earnings and consumption growth rates (intertemporal macroeconomic risk). When the representative agent is averse to the delayed resolution of intertemporal risk ($\gamma > 1/\psi$), she dislikes intertemporal macroeconomic risk, which is reflected in an increased risk-adjustment factor. Finally observe that is crucial to consider a dynamic economy in order to generate realistic magnitudes for credit spreads, as we discuss in the paper, Section IV.B.3, page 23.

# S-IX Additional results for A, BB, and B rated debt

We also investigate whether our model can correctly capture the term structure of the spread relative to AAA debt of A, BB, and B rated debt, together with the corresponding actual default probabilities.[S-1] Table S-II summarizes our results. We find that our model performs well both for safer (relative to BBB) A-rated debt and riskier BB-rated debt. For B debt, however, our model underpredicts both credit spreads and default probabilities. There are two potential explanations. A technical explanation is that empirical evidence (especially on default rates) includes callable debt. It is possible that junk firms which issue callable debt are riskier than those B-rated firms issuing non-callable debt. Most likely, however, it is because original junk issues frequently default for liquidity reasons and thus the default boundary for these firms may be somewhat higher than is optimal. Recent events from the 2008 credit crisis show that firms frequently default not because it is optimal

[S-1] For practical purposes, the AA rating is very close to the safest AAA rating, and therefore omitted. Also, there are not enough firms in the CCC rating category and below to run our matching exercise.

for equityholders to default, but because firms can not roll over their debt obligations and thus default for liquidity reasons. Arguably, the default by Lehman Brothers, the biggest default in the U.S. to date, was at least partly due to liquidity reasons. Before the credit crisis, however, it was original junk firms which were potentially more likely to experience difficulties in rolling over their debt.

## S-X Comparative statics–risk aversion and the elasticity of intertemporal substitution

In Table S-III, we summarize our results for the risk premium and leverage for relative risk aversion (RRA) of 10 and an EIS of 1.5, as in Bansal and Yaron (2004). Then in Table S-IV, in the same format, we report results for the risk premium and leverage for low EIS (Panel A), low RRA (see Panel B) and both low EIS and low RRA (Panel C). For comparison purposes, we also produce results for credit spreads.

Similar to Bansal and Yaron (2004), the unlevered risk premium drops by more than 80% to around 1% when the EIS decreases from 1.5 to 0.5 in Panel A. The reason is that the representative agent's preference for earlier resolution of intertemporal risk is diluted and as a result the risk distortion factor falls. However, the levered risk premium falls by less than the unlevered risk premium. The reduced risk-distortion factor decreases the present value of a given stream of coupon payments. Hence, it becomes cheaper for the firm to issue debt. Consequently, the firm levers up and the levered risk premium increases. In dynamics, the levered risk premium is 2.51% compared with 3.22% in the base case calibration. This is a decrease of around 30%. At the same time, value-weighted leverage increases from 28% to 39%.

Reducing RRA but holding the EIS constant has a smaller impact on the risk premium as reported in Panel B. Note that it is not possible to reduce RRA by much more, since price-earnings ratios would no longer be well-defined. As before, both the unlevered and levered risk premium fall. A decrease in RRA increases both debt and equity values, leaving leverage almost unchanged. Consequently, the levered risk premium drops significantly more compared with Panel A.

In Panel C, we study the equity market when we reduce both EIS and RRA. Obviously, both the unlevered and levered risk premia fall. Even though leverage increases relative to the base case, the levered risk premium is lower than in Panel A with high RRA. In dynamics, the levered risk premium is 1.74% compared with 3.22% in the base case.

For comparison purposes, we also produce results for credit spreads in Tables S-III and S-IV.

## S-XI Further details on the levered equity risk premium

In this section, we derive an expression for the cost of levered equity, which is extends Modigliani-Miller Proposition II (with taxes) to include cost bankruptcy and dynamic capital structure.

**Proposition S-6** *The cost of levered equity is given by*

$$\begin{aligned}\mu_{R,\nu_t} &= \mu^U_{R,\nu_t} + (1-\eta)\frac{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}(\mu^U_{R,\nu_t} - \mu^B_{R,\nu_t}) \\ &\quad + \sum_{\nu_D=1}^{2}\left[1-(1-\eta)\alpha_{\nu_D}\right]\frac{q_{D,\nu_t\nu_D}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}A_{\nu_D}(X_{D,\nu_0\nu_D})(\mu^U_{R,\nu_t} - \mu_R^{q_{D,\nu_t\nu_D}}) \\ &\quad - \sum_{\nu_U=1}^{2}\frac{q_{U,\nu_t\nu_U}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}\left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right](\mu^U_{R,\nu_t} - \mu_R^{q_{U,\nu_t\nu_U}}).\end{aligned} \tag{S-36}$$

*where $R_{\nu_0\nu_U}$ is the payment made to bondholders at refinancing, $\mu_R^{q_{D,\nu_t\nu_D}} - r_{\nu_t}$ is the risk premium on the perpetual Arrow-Debreu default claim, which pays out a unit of consumption at default if default occurs in state $\nu_D$, conditional on the current state being $\nu_t$. Similarly, $\mu_R^{q_{U,\nu_t\nu_U}} - r_{\nu_t}$ is the risk premium on the perpetual Arrow-Debreu restructuring claim, which pays out a unit of consumption at refinancing if refinancing occurs in state $\nu_U$, conditional on the current state being $\nu_t$.*

**Proof of Proposition S-6.** When capital structure is dynamic, the price of levered equity is given by

$$S_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = Div_{\nu_t}(X_t, c_{\nu_0}, \nu_0) + \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0) E_{\nu_0\nu_U},$$

where $Div_{\nu_t}$ is the present value of dividends paid to equityholders during the current refinancing period when the current state is $\nu_t$, and can be written as

$$\begin{aligned} Div_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= A_{\nu_t}(X_t) - (1-\eta)\frac{c_{\nu_0}}{r_{P,\nu_t}} + \sum_{\nu_D=1}^{2} q_{D,\nu_t\nu_D}(X_t, \nu_0)\left[(1-\eta)\frac{c_{\nu_0}}{r_{P,\nu_D}} - A_{\nu_D}(X_{D,\nu_0\nu_D})\right] \\ &\quad + \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0)\left[(1-\eta)\frac{c_{\nu_0}}{r_{P,\nu_U}} - A_{\nu_U}(X_{U,\nu_0\nu_U})\right]. \end{aligned}$$

The corporate bond price is given by

$$\begin{aligned} B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = \frac{c_{\nu_0}}{r_{P,\nu_t}} &+ \sum_{\nu_D=1}^{2} q_{D,\nu_t\nu_D}(X_t, \nu_0)\left(\alpha_{\nu_D} A_{\nu_D}(X_{D,\nu_0\nu_D}) - \frac{c_{\nu_0}}{r_{P,\nu_D}}\right) \\ &+ \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0)\left(R_{\nu_0\nu_U} - \frac{c_{\nu_0}}{r_{P,\nu_U}}\right). \end{aligned}$$

We rewrite the above expressions for prices as

$$\begin{aligned} S_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= A_{\nu_t}(X_t) - (1-\eta)c_{\nu_0}\left(\frac{1}{r_{P,\nu_t}} - \sum_{\nu_D=1}^{2} \frac{q_{D,\nu_t\nu_D}(X_t, \nu_0)}{r_{P,\nu_D}} - \sum_{\nu_U=1}^{2} \frac{q_{U,\nu_t\nu_U}(X_t, \nu_0)}{r_{P,\nu_U}}\right) \\ &\quad - \sum_{\nu_D=1}^{2} q_{D,\nu_t\nu_D}(X_t, \nu_0) A_{\nu_D}(X_{\nu_0\nu_D}) - \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0) A_{\nu_U}(X_{U,\nu_0\nu_U}) \\ &\quad + \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0) E_{\nu_0\nu_U}, \end{aligned}$$

and

$$\begin{aligned} B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= c_{\nu_0}\left(\frac{1}{r_{P,\nu_t}} - \sum_{\nu_D=1}^{2} \frac{q_{D,\nu_t\nu_D}(X_t, \nu_0)}{r_{P,\nu_D}} - \sum_{\nu_U=1}^{2} \frac{q_{U,\nu_t\nu_U}(X_t, \nu_0)}{r_{P,\nu_U}}\right) \\ &\quad + \sum_{\nu_D=1}^{2} q_{D,\nu_t\nu_D}(X_t, \nu_0)\alpha_{\nu_D} A_{\nu_D}(X_{\nu_0\nu_D}) + \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0) R_{\nu_0\nu_U}. \end{aligned}$$

Since levered firm value is given by $F_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = S_{\nu_t}(X_t, c_{\nu_0}, \nu_0) + B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$, it then follows that

$$\begin{aligned} F_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= A_{\nu_t}(X_t) + \eta B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) - \sum_{\nu_D=1}^{2} \left[1 - (1-\eta)\alpha_{\nu_D}\right] q_{D,\nu_t\nu_D}(X_t, \nu_0) A_{\nu_D}(X_{D,\nu_0\nu_D}) \\ &\quad + \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0) \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right]. \end{aligned}$$

Hence,

$$\begin{aligned} S_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= F_{\nu_t}(X_t, c_{\nu_0}, \nu_0) - B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) \\ &= A_{\nu_t}(X) - (1-\eta) B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) - \sum_{\nu_D=1}^{2} \left[1 - (1-\eta)\alpha_{\nu_D}\right] q_{D,\nu_t\nu_D}(X_t, \nu_0) A_{\nu_D}(X_{D,\nu_0\nu_D}) \\ &\quad + \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0) \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right]. \end{aligned}$$

Applying Ito's Lemma gives

$$\begin{aligned} \frac{dS_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} &= \frac{A_{\nu_t}(X)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \frac{dA_{\nu_t}(X)}{A_{\nu_t}(X)} - (1-\eta) \frac{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \frac{dB_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \\ &\quad - \sum_{\nu_D=1}^{2} \left[1 - (1-\eta)\alpha_{\nu_D}\right] \frac{q_{D,\nu_t\nu_D}(X_t, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \frac{dq_{D,\nu_t\nu_D}(X_t, \nu_0)}{q_{D,\nu_t\nu_D}(X_t, \nu_0)} A_{\nu_D}(X_{D,\nu_0\nu_D}) \\ &\quad + \sum_{\nu_U=1}^{2} \frac{q_{U,\nu_t\nu_U}(X_t, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \frac{dq_{U,\nu_t\nu_U}(X_t, \nu_0)}{q_{U,\nu_t\nu_U}(X_t, \nu_0)} \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right]. \end{aligned}$$

Observe that

$$\begin{aligned} \frac{A_{\nu_t}(X)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} &= 1 + (1-\eta) \frac{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \\ &\quad + \frac{\sum_{\nu_D=1}^{2} \left[1 - (1-\eta)\alpha_{\nu_D}\right] q_{D,\nu_t\nu_D}(X_t, \nu_0) A_{\nu_D}(X_{D,\nu_0\nu_D})}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \\ &\quad - \frac{\sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0) \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right]}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}. \end{aligned}$$

Hence,

$$
\begin{aligned}
\frac{dS_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \;=\;& \Bigg(1 + \frac{\sum_{\nu_D=1}^{2} \left[1-(1-\eta)\alpha_{\nu_D}\right] q_{D,\nu_t\nu_D}(X_t,\nu_0) A_{\nu_D}(X_{D,\nu_0\nu_D})}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \\
& - \frac{\sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t,\nu_0) \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right]}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}\Bigg) \frac{dA_{\nu_t}(X)}{A_{\nu_t}(X)} \\
& + (1-\eta)\frac{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \left(\frac{dA_{\nu_t}(X)}{A_{\nu_t}(X)} - \frac{dB_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}\right) \\
& - \sum_{\nu_D=1}^{2} \left[1-(1-\eta)\alpha_{\nu_D}\right] \frac{q_{D,\nu_t\nu_D}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \frac{dq_{D,\nu_t\nu_D}(X_t,\nu_0)}{q_{D,\nu_t\nu_D}(X_t,\nu_0)} A_{\nu_D}(X_{D,\nu_0\nu_D}) \\
& + \sum_{\nu_U=1}^{2} \frac{q_{U,\nu_t\nu_U}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \frac{dq_{U,\nu_t\nu_U}(X_t,\nu_0)}{q_{U,\nu_t\nu_U}(X_t,\nu_0)} \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right].
\end{aligned}
$$

Therefore, the levered risk premium is given by

$$
\begin{aligned}
\mu_{R,\nu_t} - r_{\nu_t} \;=\;& \Bigg(1 + \frac{\sum_{\nu_D=1}^{2} \left[1-(1-\eta)\alpha_{\nu_D}\right] q_{D,\nu_t\nu_D}(X_t,\nu_0) A_{\nu_D}(X_{D,\nu_0\nu_D})}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \\
& - \frac{\sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t,\nu_0) \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_D}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right]}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}\Bigg) (\mu^U_{R,\nu_t} - r_{\nu_t}) \\
& + (1-\eta)\frac{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} (\mu^U_{R,\nu_t} - \mu^B_{R,\nu_t}) \\
& - \sum_{\nu_D=1}^{2} \left[1-(1-\eta)\alpha_{\nu_D}\right] \frac{q_{D,\nu_t\nu_D}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} A_{\nu_D}(X_{D,\nu_0\nu_D}) (\mu_R^{q_{D,\nu_t\nu_D}} - r_{\nu_t}) \\
& + \sum_{\nu_U=1}^{2} \frac{q_{U,\nu_t\nu_U}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right] (\mu_R^{q_{U,\nu_t\nu_U}} - r_{\nu_t}),
\end{aligned}
$$

which simplifies to

$$
\begin{aligned}
\mu_{R,\nu_t} \;=\;& \mu^U_{R,\nu_t} + (1-\eta)\frac{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} (\mu^U_{R,\nu_t} - \mu^B_{R,\nu_t}) \\
& - \sum_{\nu_D=1}^{2} \left[1-(1-\eta)\alpha_{\nu_D}\right] \frac{q_{D,\nu_t\nu_D}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} A_{\nu_D}(X_{D,\nu_0\nu_D}) (\mu_R^{q_{D,\nu_t\nu_D}} - \mu^U_{R,\nu_t}) \\
& + \sum_{\nu_U=1}^{2} \frac{q_{U,\nu_t\nu_U}(X_t,\nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} \left[(1-\eta)R_{\nu_0\nu_U} - A_{\nu_U}(X_{U,\nu_0\nu_U}) + E_{\nu_0\nu_U}\right] (\mu_R^{q_{U,\nu_t\nu_U}} - \mu^U_{R,\nu_t}).
\end{aligned}
$$

■

First, observe that if bankruptcy and refinancing cannot occur, then (S-36) reduces to (44). Second, observe that if capital structure is static, then (S-36) reduces to

$$\mu_{R,\nu_t} = \mu^U_{R,\nu_t} + (1-\eta)\frac{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}(\mu^U_{R,\nu_t} - \mu^B_{R,\nu_t}) \tag{S-37}$$

$$- \sum_{\nu_D=1}^{2} [1-(1-\eta)\alpha_{\nu_D}] \frac{q_{D,\nu_t\nu_D}(X_t, \nu_0)}{S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} A_{\nu_D}(X_{D,\nu_0\nu_D})(\mu_R^{q_{D,\nu_t\nu_D}} - \mu^U_{R,\nu_t}). \tag{S-38}$$

Since Arrow-Debreu default claims pay off if default (a bad outcome) occurs, then $\mu_R^{q_{D,\nu_t\nu_D}} < 0$, which implies $\mu^U_{R,\nu_t} - \mu_R^{q_{D,\nu_t\nu_D}}$, and so the above formula shows that *if leverage is kept fixed*, the default option increases the equity risk premium. Similarly, by comparing (S-36) with (44), observe that the possibility of refinancing decreases the levered equity risk premium. The latter results are valid *provided leverage does not change*. We hence refer to these results as **direct effects**. However, in our model leverage is chosen optimally, and so leverage does change when the default and refinancing options are introduced, leading to **indirect effects** on the levered risk premium. With the option to default, leverage is reduced, which reduces the levered risk premium. This indirect effect dominates the direct effect, leading to an overall decrease in the levered risk premium. With the introduction of the refinancing option, leverage is reduced, leading to a fall in the risk premium. Thus, both the direct and indirect effects lead to a fall in the risk premium. In fact, if we hold leverage constant and compare the levered risk premium with and without the refinancing option, there is very little change, implying that the indirect effect is the more quantitatively important.

# S-XII The risk adjustment in a three state model

In this section, we extend the model to include three states. The presence of a middle state allows us to investigate what happens when there is a state, where the economy can both improve and worsen. Table S-V shows the parameter values for the 3 state version of our model. Table S-VI compares risk adjustments in the 2 and 3 state models under static capital structure. Panel A – C show that risk adjustments are still procyclical on the 3 state model. Furthermore, the magnitudes of the risk adjustments in the 3 state model are very close to those in the 2 state model shown in Panels D – E.

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**Table S-I : Model comparison**

This table provides a comparison between stripped down versions of our model. In Model 1 there is no intertemporal risk, the first and second moments of consumption and earnings growth rates do not switch. In Model 2 the first and second moments of earnings growth switch but the first and second moments of consumption growth do not. In Model 3 the first and second moments of both earnings and consumption growth switch. In all models, capital structure is dynamic. Results in Panel A are given at the refinancing date for an individual firm, whereas in Panel B results are for a dynamic cross-section of firms. We assume $\psi = 0.75$ to ensure that price-earnings ratios in Model 1 are positive. In Panel A, we consider an individual firm at refinancing point. In Panel B, we consider a dynamic cross-section. The credit spread is an equally weighted-average across firms in the economy, whereas the risk premium is value-weighted average.

| Model | | | 1 | 2 | 3 |
|---|---|---|---|---|---|
| Risk aversion | $\gamma$ | | 10.00 | 10.00 | 10.00 |
| EIS | $\psi$ | | 0.75 | 0.75 | 0.75 |
| Distortion factor | $\omega$ | | 1.00 | 1.00 | 1.38 |
| **Panel A: Individual Firm at Refinancing Point** | | | | | |
| 10-year credit spread | $s$ | b.p. | 28.18 | 37.97 | 55.36 |
| 10-year actual default probability | $p_D$ | % | 4.79 | 5.68 | 5.05 |
| 10-year risk adjustment | $\mathcal{R}$ | | 1.05 | 1.07 | 1.62 |
| 10-year time adjustment | $\mathcal{T}$ | | 0.68 | 0.69 | 0.75 |
| Levered risk premium | $\mu_R - r$ | % | 0.33 | 0.35 | 2.53 |
| **Panel B: Dynamic Cross Section** | | | | | |
| 10-year credit spread | $s$ | b.p. | 113.31 | 129.77 | 157.62 |
| 10-year actual default probability | $p_D$ | % | 14.60 | 15.03 | 13.88 |
| 10-year risk adjustment | $\mathcal{R}$ | | 1.03 | 1.02 | 1.31 |
| 10-year time adjustment | $\mathcal{T}$ | | 0.78 | 0.78 | 0.81 |
| Levered risk premium | $\mu_R - r$ | % | 0.35 | 0.37 | 2.83 |

**Table S-II : Benchmark results for various ratings**

This table reports model-implied results under 'true dynamics' for credit spreads and actual default probabilities for four ratings, A, BBB, BB, and B, and for five-year and ten-year horizons. Results for BBB are taken from Panel C, Table III of the paper. All results are obtained in the same way as for the BBB sample as discussed in the paper. Empirical evidence on credit spreads is from Davydenko and Strebulaev (2007) (for all maturities). Empirical data on historical default rates is from Moody's over the period of 1920–2007 (see Cantor et al. (2008)).

| | | Units | A | BBB | BB | B |
|---|---|---|---|---|---|---|
| Credit spread, True dynamics | $s$ | | | | | |
| 5 years | | b.p. | 34 | 75 | 142 | 259 |
| 10 years | | b.p. | 56 | 115 | 186 | 280 |
| Credit spread, Empirical | $s$ | | | | | |
| 5 years | | b.p. | 39 | 77 | 182 | 374 |
| 10 years | | b.p. | 31 | 72 | 188 | 342 |
| Actual default probability, True dynamics | $p_D$ | | | | | |
| 5 years | | % | 0.76 | 3.46 | 6.61 | 12.21 |
| 10 years | | % | 3.84 | 8.38 | 15.66 | 23.56 |
| Default probability, Empirical | $p_D$ | | | | | |
| 5 years | | % | 1.12 | 3.14 | 9.59 | 21.43 |
| 10 years | | % | 2.90 | 7.06 | 18.44 | 33.93 |

**Table S-III : Equity risk premium, leverage and credit spreads, base case**

This table presents results for the risk premium, leverage and the 10-year credit spread for an elasticity of intertemporal substitution (EIS) of 1.5 and relative risk aversion (RRA) of 10. Results for the risk premium and leverage are for the model with optimal coupon, default and restructuring decisions. For the dynamic cross-section, we generate 1,000 economies each containing 3,000 firms over 100 years at quarterly frequency. The statistics are averaged over data sets. Results for the 10-year credit spread are produced for BBB debt in the same way as in Panels B and C of Table III of the paper, i.e. the first two results are for the specification with optimal default boundary and exogenous leverage and the third result is for a dynamic simulated cross-section of BBB firms implied by the model with optimal default and dynamic capital structure decisions. Simulation procedure is described in Appendix C of the paper.

| | | Units | Static Capital Structure Initial Date | Dynamic Capital Structure Refinancing Date | Dynamic Capital Structure True Dynamics |
|---|---|---|---|---|---|
| Risk-free rate | $r$ | % | 2.97 | 2.97 | 2.97 |
| Unlevered Equity Premium | $\mu_R - r$ | % | 1.91 | 1.91 | 1.91 |
| Levered equity premium | $\mu_R - r$ | % | 3.08 | 2.66 | 3.22 |
| 25% quantile | | % | | | 2.91 |
| 75% quantile | | % | | | 3.40 |
| Equally-weighted leverage | $B/(B+S)$ | % | 42.52 | 28.74 | 40.56 |
| Value-weighted leverage | $B/(B+S)$ | % | 42.52 | 28.74 | 28.10 |
| Market-wide leverage | $B/(B+S)$ | % | 42.52 | 28.74 | 33.87 |
| Credit spread, 10-year | $s$ | b.p. | 49 | 42 | 115 |

**Table S-IV : Equity risk premium, leverage and credit spreads, comparative statics**

This table presents results for the risk premium, leverage and credit spreads for low EIS in Panel A, low RRA in Panel B and low EIS and low RRA in Panel C.

| | | Units | Static Capital Structure Initial Date | Dynamic Capital Structure Refinancing Date | Dynamic Capital Structure True Dynamics |
|---|---|---|---|---|---|
| Panel A: Low EIS ($\psi = 0.5$ and $\gamma = 10$) | | | | | |
| Risk-free rate | $r$ | % | 7.45 | 7.45 | 7.45 |
| Unlevered Equity Premium | $\mu_R - r$ | % | 1.04 | 1.04 | 1.04 |
| Levered equity premium | $\mu_R - r$ | % | 2.49 | 2.07 | 2.51 |
| 25% quantile | | % | | | 2.28 |
| 75% quantile | | % | | | 2.59 |
| Equally-weighted leverage | $B/(B+S)$ | % | 53.34 | 41.17 | 52.15 |
| Value-weighted leverage | $B/(B+S)$ | % | 53.34 | 41.17 | 39.29 |
| Market-wide leverage | $B/(B+S)$ | % | 53.34 | 41.17 | 47.16 |
| Credit spread, 10-year | $s$ | b.p. | 43 | 38 | 105 |
| Panel B: Low RRA ($\psi = 1.5$ and $\gamma = 7.5$) | | | | | |
| Risk-free rate | $r$ | % | 3.02 | 3.02 | 3.02 |
| Unlevered Equity Premium | $\mu_R - r$ | % | 1.38 | 1.38 | 1.38 |
| Levered equity premium | $\mu_R - r$ | % | 2.44 | 1.88 | 1.95 |
| 25% quantile | | % | | | 1.83 |
| 75% quantile | | % | | | 2.02 |
| Equally-weighted leverage | $B/(B+S)$ | % | 47.66 | 26.76 | 41.11 |
| Value-weighted leverage | $B/(B+S)$ | % | 47.66 | 26.76 | 24.46 |
| Market-wide leverage | $B/(B+S)$ | % | 47.66 | 26.76 | 28.44 |
| Credit spread, 10-year | $s$ | b.p. | 39 | 33 | 93 |
| Panel C: Low EIS and low RRA ($\psi = 0.5$ and $\gamma = 7.5$) | | | | | |
| Risk-free rate | $r$ | % | 7.42 | 7.42 | 7.42 |
| Unlevered Equity Premium | $\mu_R - r$ | % | 0.72 | 0.72 | 0.72 |
| Levered equity premium | $\mu_R - r$ | % | 1.77 | 1.44 | 1.74 |
| 25% quantile | | % | | | 1.60 |
| 75% quantile | | % | | | 1.87 |
| Equally-weighted leverage | $B/(B+S)$ | % | 54.84 | 41.89 | 52.57 |
| Value-weighted leverage | $B/(B+S)$ | % | 54.84 | 41.89 | 39.98 |
| Market-wide leverage | $B/(B+S)$ | % | 54.84 | 41.89 | 47.86 |
| Credit spread, 10-year | $s$ | b.p. | 36 | 32 | 86 |

**Table S-V : Parameter estimates, three state model**

This table reports conditonal estimates of model parameters for a 3-state model. To calibrate the model to the aggregate US economy, quarterly real non-durable plus service consumption expenditure from the Bureau of Economic Analysis and quarterly earnings data from Standard and Poor's, provided by Robert J. Shiller, are used. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. The estimates of consumption growth rate and volatility, earnings growth rate and volatility, and correlation between earnings and consumption are obtained by maximum likelihood and based on quarterly log growth rates for the period from 1947 to 2005. All variables are given per annum and in per cent (0.01 means 1% p.a.)

| Conditional estimates | | | | |
|---|---|---|---|---|
| Parameter | Symbol | State 1 | State 2 | State 3 |
| Consumption growth rate | $g_i$ | 0.0171 | 0.0321 | 0.0471 |
| Consumption growth volatility | $\sigma_{C,i}$ | 0.0112 | 0.0101 | 0.0090 |
| Earnings growth rate | $\theta_i$ | -0.0274 | 0.0361 | 0.0997 |
| Earnings growth volatility | $\sigma^s_{X,i}$ | 0.1280 | 0.1012 | 0.0743 |
| Idiosyncratic earnings growth volatility | $\sigma^{id}_X$ | 0.2258 | 0.2258 | 0.2258 |
| Correlation | $\rho_{XC}$ | 0.1998 | 0.1998 | 0.1998 |
| Actual long-run probabilities | $f_i$ | 0.3333 | 0.3333 | 0.3333 |
| Actual convergence rate to long-run | $p$ | 0.7646 | 0.7646 | 0.7646 |
| Annual discount rate | $\beta$ | 0.01 | 0.01 | 0.01 |
| Tax rate | $\eta$ | 0.15 | 0.15 | 0.15 |
| Bankruptcy costs | $1 - \alpha_i$ | 0.30 | 0.20 | 0.10 |
| Debt issuance cost | $\iota_i$ | 0.03 | 0.02 | 0.01 |

**Table S-VI : Risk adjustments**

This table shows perpetual risk adjustments, $\mathcal{R}_{\nu_t,\nu_D}$, at the initial date when capital structure is static. Panels A, B and C report values for a 3 state economy when the initial state of the economy is 1, 2 and 3, respectively. Panels D and E report values for a 2 state economy when the initial state of the economy is 1 and 2, respectively.

Panel A: initial state is 1

| | DEFAULT STATE, $\nu_D$ | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| CURRENT STATE, $\nu_t$ | | | |
| 1 | 1.416 | 0.727 | 0.6281 |
| 2 | 1.436 | 0.738 | 0.637 |
| 3 | 1.454 | 0.747 | 0.645 |

Panel B: initial state is 2

| | DEFAULT STATE, $\nu_D$ | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| CURRENT STATE, $\nu_t$ | | | |
| 1 | 1.387 | 0.713 | 0.616 |
| 2 | 1.407 | 0.723 | 0.624 |
| 3 | 1.425 | 0.732 | 0.632 |

Panel C: initial state is 3

| | DEFAULT STATE, $\nu_D$ | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| CURRENT STATE, $\nu_t$ | | | |
| 1 | 1.376 | 0.707 | 0.610 |
| 2 | 1.396 | 0.718 | 0.619 |
| 3 | 1.414 | 0.727 | 0.627 |

Panel D: initial state is 1

| | DEFAULT STATE, $\nu_D$ | |
|---|---|---|
| | 1 | 2 |
| CURRENT STATE, $\nu_t$ | | |
| 1 | 1.472 | 0.739 |
| 2 | 1.509 | 0.757 |

Panel E: initial state is 2

| | DEFAULT STATE, $\nu_D$ | |
|---|---|---|
| | 1 | 2 |
| CURRENT STATE, $\nu_t$ | | |
| 1 | 1.434 | 0.721 |
| 2 | 1.470 | 0.738 |